Six Sigma Quality: Concepts & Cases- Volume I
STATISTICAL TOOLS IN SIX SIGMA DMAIC PROCESS WITH MINITAB® APPLICATIONS

Chapter 7

Measurement System Analysis
Gage Repeatability & Reproducibility (Gage R&R) Study

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Chapter Highlights

The chapter discusses the importance of measurement and measurement system analysis (MSA) in Six Sigma. It is critical to assess the accuracy of the measurement process before collecting data. Overlooking the measurement process can be expensive as it may divert the effort in fixing the wrong problem. This chapter deals with the following concepts related to measurement system.

1. *Terms Related to the Measurement Systems Analysis : Systematic Errors, Random Errors, Metrology, Gage, Bias, and Resolution*
2. *Accuracy, Precision, Repeatability, and Reproducibility*
3. *Graphical Analysis of Gage Study: Gage Run Charts*
4. *Quantitative methods of Gage analysis - Examples*
5. *Analytical Gage Study: Gage R & R*
6. *Elements of the Measurement Process: equipment, operators, and parts*
7. *Gage Repeatability and Reproducibility (Gage R&R) study with cases*
8. *Computer analysis of gage study including*

    Gage R&R Study (Crossed) – X-bar/R Method and ANOVA
    Gage R & R Study (Nested)
    Gage Linearity and Bias Study
    Attribute Gage Study (Analytical Method)

Chapter Outline

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This sample file contains explanation and numerous examples on measurement system analysis including Gage R & R from Chapter 7 of our Six Sigma Volume 1. For detailed treatment of Gage R & R see Chapter 7 of the book. The book contains numerous cases, examples, step-wise computer instructions with data files.

Introduction

The conclusions drawn from the statistical methods depend on the accuracy of data. If the measuring instrument and the measurement method are not capable of making accurate and repeatable measurements, the data can have significant measurement error. In such cases, the conclusions drawn from the data are inaccurate and misleading. It is critical to assess the accuracy of the measurement process at the start of the study. Inaccurate measurements may lead to false signals on control charts. In the presence of significant error in the measurement process, a capable process may be confused with an incapable process. Overlooking the measurement process can be expensive by diverting the effort in fixing the wrong problem. When the major source of variation is from the measurement process, significant time and money can be wasted in fixing and controlling the process. Several factors affect the reliability of measurements including:

- differences in measurement procedures,
- differences among operators,
- instrument repeatability and reproducibility, and
- instrument calibration and resolution.

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Figure 7.1 shows the measurement errors and their causes. This chapter is concerned with the analysis of measurement systems including repeatability, reproducibility, bias, stability, and linearity.

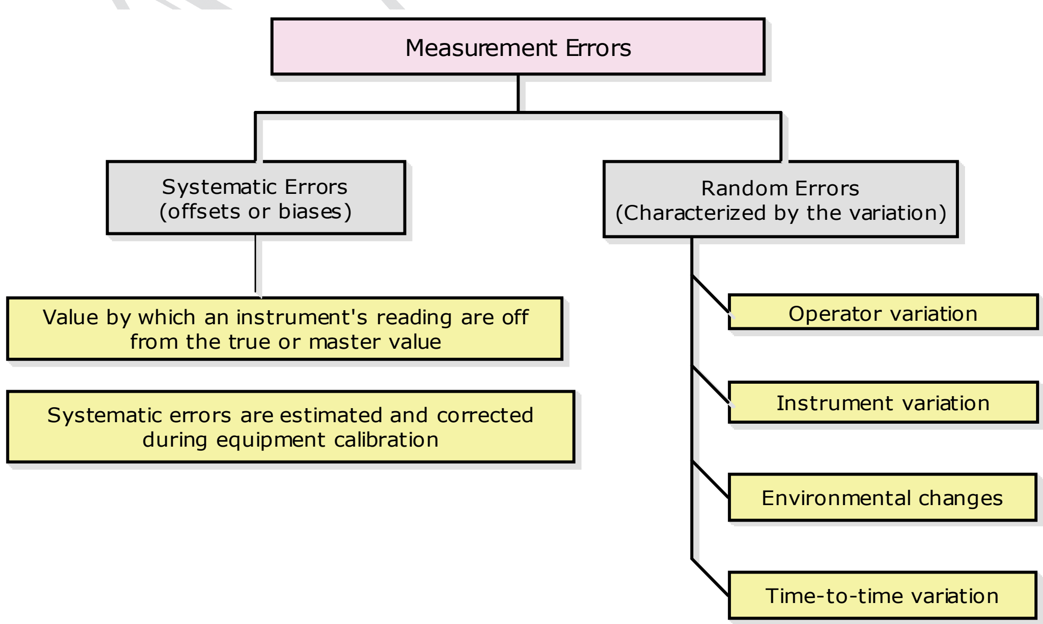

Figure 7.1: Measurement Errors

When measurements are made, some of the observed variability is due to the part or the product being measured, and some variability can be attributed to measurement or gage variability. The total variability can be stated as:

$$\sigma^2_{Total} = \sigma^2_{product} + \sigma^2_{measurement}$$

Where, σ^2_{Total} is total variation, $\sigma^2_{product}$ is the component of variation due to the product or the part, $\sigma^2_{measurement}$ is the component of variation due to measurement error. Note: $\sigma^2_{measurement}$ is also referred as σ^2_{gage}.

The measurement system analysis is commonly known as ***Gage Repeatability and Reproducibility (Gage R&R)*** study. The purpose of measurement system analysis or Gage R&R study is to determine the part of variation in the data resulting from the variation in the measurement system. This chapter presents statistical methods that are used to separate the components of variation in the measurement process and assess the gage capability.

Terms Related to Measurement Systems

Systematic Errors (or offsets)*:* These errors are defined as the constant values by which a measurement instrument's readings are off from the true or reference value (or a master value).

Random Errors*:* These are measurement errors caused by differences among operators, differences among the measuring equipments, differences over time, or the differences due to change in the environmental conditions….

Metrology:

Gage: Gages are devices of preset dimensions used to compare product dimensions to check whether the product meets or exceeds specifications.
Bias: It is the difference between the average of measurements and the true or reference value of the part. The reference value is also known as the *master value.*

Resolution*:* The resolution of measurements refers to the number of digits of precision needed of the measured value.

$$CV = \frac{s}{x} or, CV = \frac{\sigma}{\mu}$$

:
: ***continued…***

Accuracy, Precision, Repeatability, and Reproducibility

To assess the measurement errors, the concepts of accuracy, precision, repeatability, and reproducibility should be clearly understood.

ACCURACY AND PRECISION

Measurement system errors can be divided into two categories: accuracy and precision.

Accuracy is the difference between the average of measurements made on a part and the true value of that part or,

$$\text{Accuracy} = \overline{x}_m - x$$

Precision is the degree to which repeated measurements tend to agree with each other. It is getting consistent results repeatedly.

Accuracy refers to long-term average of measurements while precision refers to long-term variation.

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The difference between the precision and accuracy can be seen in Figure 7.2.

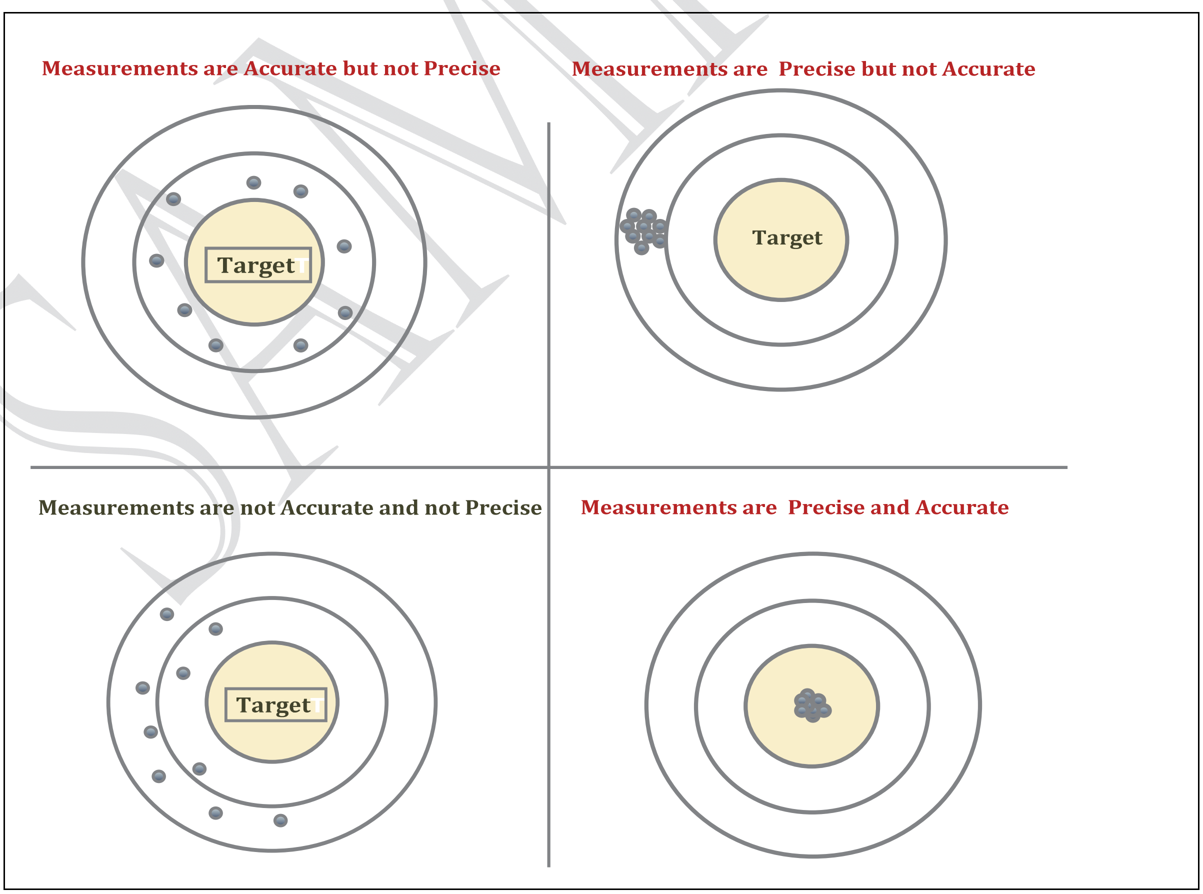


Figure 7.2: Accuracy and Precision

Measurement error is estimated using accuracy and precision. Accuracy and precision of a measurement system are broken down into components shown below in Figures 7.2 and 7.3.

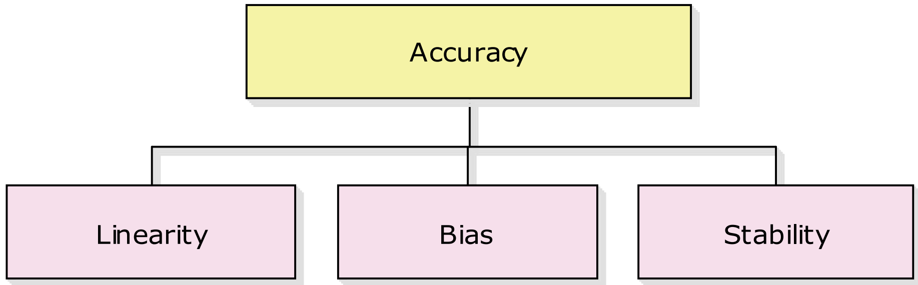


Figure 7.2: Components of Accuracy

Gage linearity is ……………

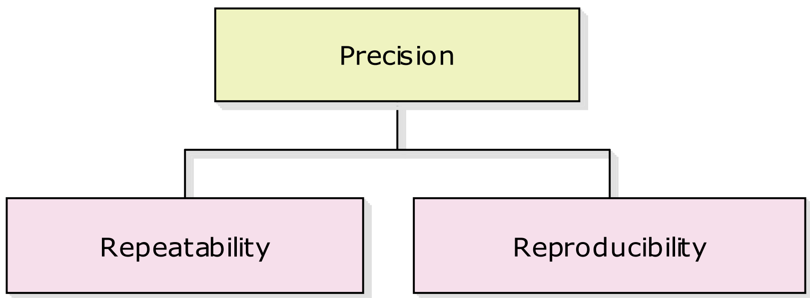


Figure 7.3: Components of Precision

REPEATABILITY:

………., *repeatability of a measuring instrument refers to how well the instrument is repeatedly able to measure the same characteristic under the same condition.*

Example*:* The repeatability of a measuring instrument is to be determined. An operator measured the length of a standard GAGE block of 0.500 inches 10 times. The measured values are shown below.

Length (in.)

0.498 0.500 0.502 0.500 0.500 0.500 0.501 0.502 0.499 0.500

The mean and standard deviation for these measurements are $\bar{x}_m = 0.5002$ and $s_m = 0.00119$. The true length of the gage block is 0.500 which is constant. The accuracy and precision of the instrument are

Accuracy = average of the measurements - true value or,

$$Accuracy = \bar{x}_m - x = 0.5002 - 0.500 = +0.0002$$

The accuracy of + 0.0002 means that…

The six-sigma (6σ) precision, based on the normal distribution is $6S_m$.
In the above example the measurements …………….

The stability or the drift

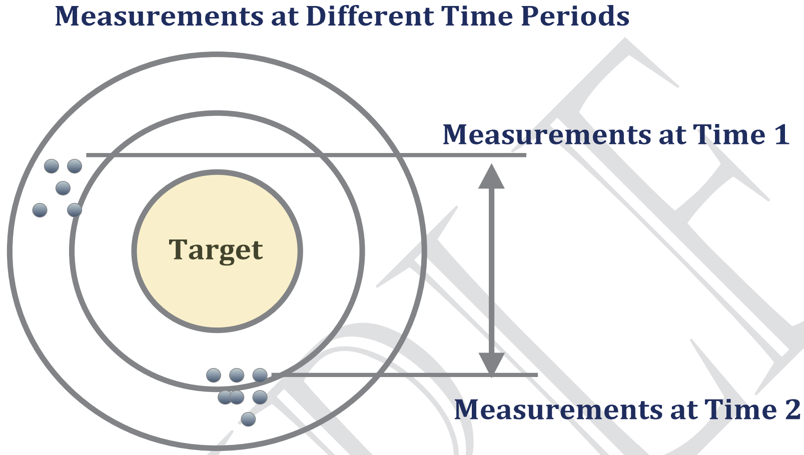


Figure 7.4: Measurements Made at Different Time Periods

REPRODUCIBILITY:

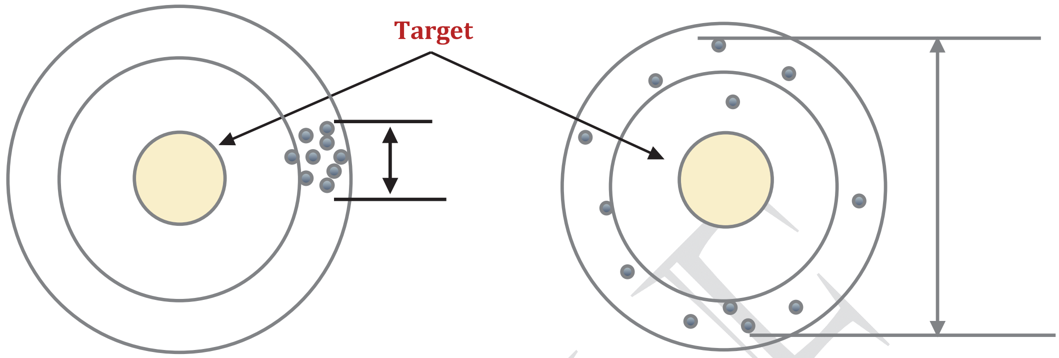
………….reproducibility is the variation due to different operators using the same measuring instrument at different time periods, and different environmental conditions.

Estimating Measurement Error: Some Measurement Models

A simple measurement model can be written as

$$x_m = x + \varepsilon \qquad (7.1)$$

Where x_m is the measured value, x is the 'true' or master value, and ε is the measurement error. "The master value is the measurement made with the most accurate instrument" (AIAG Manual). Equation (7.1) can be modified to include error terms such as the measurement instrument error, part error, ………………..



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Continued…

Examples:

Example7.2

The quality improvement team involved in establishing the process capability of a quality characteristic is ready to measure the required characteristic. Before collecting the data and performing the measurements, the team would like to get an assessment of gage capability of the instrument to be used to perform actual measurements. The operator responsible for performing the actual measurements selected 25 parts and measured each part twice. The data are shown in Table 7.2.

(a) Using a computer package, construct and analyze $\bar{x}$ and R control charts for the data in Table 7.2? What conclusions you can draw about the use of the gage from the charts?

(b) Determine the gage capability of the measuring instrument. The specification limits are: USL=70 and LSL = 10. Determine the P/T ratio. What can you say about the gage capability?

Solution: (a) To determine the gage capability, we first construct the $\bar{x}$ and R charts for the measurement data. These control charts are shown in Figure 7.11. The charts can be interpreted in the following way.

$\bar{x}$ ***Chart:*** The $\bar{x}$ chart has a slightly different interpretation in this case. …… As a general rule, the measuring increments should be about 0.1 of the accuracy required in the final….

R-Chart: The R-chart is in control and shows the gage capability, or …

The R values show the difference between the measurements made on the same part. The out -of –control points on the R chart is an indication that the ……… measurements. For our example, the R-chart is in control (see Figure 7.11). This is an indication that ……..

Table 7.2: Two-measurements of the Same Part
(M1: Measurement 1, M2: Measurement 2)

Part No.	M1	M2	$\bar{x}$	Range (R)
1	20	19	19.5	1
2	19	18	18.5	1
3	23	22	22.5	1
4	20	20	20.0	0
:				
:				
23	23	23	23.0	0
24	24	23	23.5	1
25	22	21	21.5	1
			-------	-------

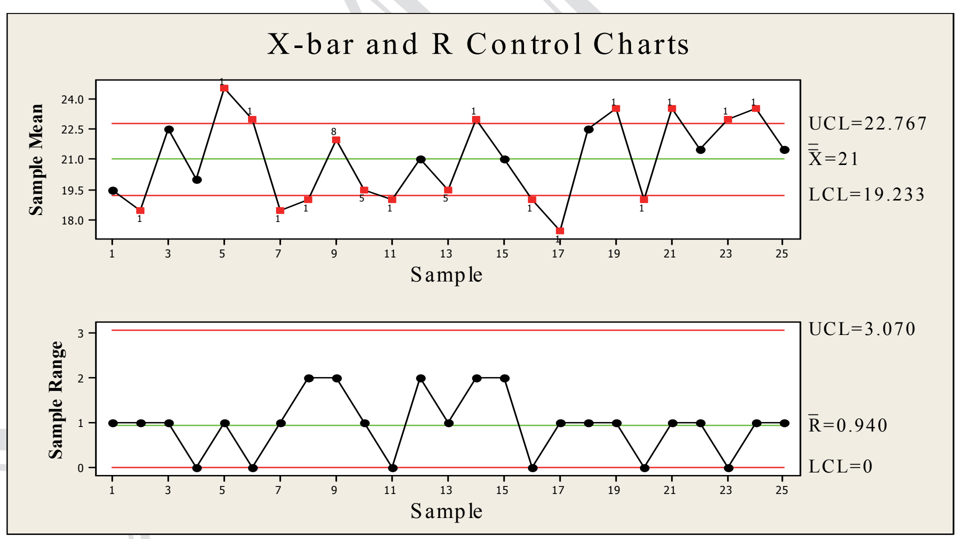

Figure 7.11: Control Charts for the Measurement Data in Table 7.2

(a) To determine the gage capability, we first calculate the following:

Standard deviation of measurement error,

$$\hat{\sigma}_{gage} = \frac{\overline{R}}{d_2} = 0.851$$

(Note: d_2 is a factor that depends on the subgroup size ….

The estimate of gage capability,

$$6\hat{\sigma}_{gage} = 5.11$$

This means that the individual measurements can vary as much as

$$\pm 3\hat{\sigma}_{gage}$$

(Note: the estimate of the gage capability is based on the assumption that the measurements errors are normally distributed).

The precision-to-tolerance or, P/T ratio,

$$\frac{P}{T} = \ldots = 0.085$$

The value of P/T ratio of 0.088 (less than 0.1), is an indication of ……

Example 7.3

Use the data in Example 7.2 to estimate the total variability, and the variability due to the part. What percentage of the total variability is due to the gage?

Solution:

To estimate the total variability, calculate the standard deviation of the sample measurements of the data …

Therefore, the total variability,

$$\hat{\sigma}^2_{Total} = \ldots = 4.33$$

:

:

:

$$\hat{\sigma}_{gage} = 0.851$$

We can now calculate the standard deviation of part variation as,

$$\hat{\sigma}_{part} = \sqrt{3.60} = 1.90$$

Therefore, the total variability due to the gage as percent of part variability ….is 44.8%

:

:

The above value represents the gage capability. Unlike the gage capability using P/T ratio, this method does not require the specification limits. Therefore, the above ratio is more meaningful in many cases.

Example7.4 : Assessing Measurement Errors: Repeatability and Reproducibility

A six sigma quality improvement team wants to assess the components of measurement errors before establishing the process capability for a product characteristic. They want to estimate the two components of measurement errors – repeatability and reproducibility. Three operators were selected to perform the actual measurements on the selected 25 parts. Each operator used the same gage to measure 25 parts two times each. The data are shown in Table 7.3.

(a) Estimate the gage repeatability and reproducibility.

(b) Estimate the standard deviation of measurement error.

(c) If the specification limits are 70±10, what can you say about the gage capability?

Table 7.3: Data to Determine the Measurement Errors
(Each Operator Takes Two Measurements: M1 and M2)

	Operator 1				**Operator 2**				**Operator 3**			
No.	**M1**	**M2**	$\bar{x}$	**R**	**M1**	**M2**	$\bar{x}$	**R**	**M1**	**M2**	$\bar{x}$	**R**
1	20	19	19.5	1	21	20	20.5	1	19	19	19.0	0
2	19	18	18.5	1	23	23	23.0	0	23	22	22.5	1
3	23	22	22.5	1	19	18	18.5	1	18	20	19.0	2
4	:				:				24	22	23.0	2
5	:				:				20	19	19.5	1
6	:								18	18	18.0	0
7									20	18	19.0	2
8									:			
9									:			
10									:			
11												
12												
13												
14	:											
15	:											
16	:											
17												
18	23	22	22.5	1								
19	24	23	23.5	1								
20	19	19	19.0	0								
21	24	23	23.5	1	24	22	23.0	2				
22	22	21	21.5	1	21	20	20.5	1				
23	23	23	23.0	0	22	21	21.5	1				
24	24	23	23.5	1	23	22	22.5	1				
25	22	21	21.5	1	19	19	19.0	0				
			$\bar{\bar{x}}_1$	↓ $\bar{R}_1 = 0.960$				↓ $\bar{R}_2 =$			$\bar{\bar{x}}_3$	↓ $\bar{R}_3 =$

Solution:

Note that the measurement error is defined as

$$\sigma^2_{measurementerror} = \sigma^2_{gage} = \sigma^2_{repeatability} + \sigma^2_{reproducibility}$$

Where, *repeatability of a measuring instrument refers to how well the instrument is repeatedly able to measure the same characteristic under the same condition.*

Reproducibility is the variation due to different operators using the same measuring instrument at different time periods, and different environmental conditions.

Estimating Gage Repeatability:

When $\bar{x}$ and R charts are used, the estimate of gage repeatability is calculated using the following formula:

$$\hat{\sigma}_{repeatability} = \frac{\bar{\bar{R}}}{d_2}$$

:
:
:

The value of $\bar{\bar{R}}$ can be calculated using the average of the three average ranges in Table7.3
For our example,

$$\bar{\bar{R}} = \frac{1}{3}(0.960 + 1.080 + 1.160) = 1.067$$

and d_2 = 1.128 for subgroup size of two (each operator measured the part twice). Using these values,

Estimating Gage Reproducibility:

The gage reproducibility is the variability due to the three operators in the study. If the $\bar{\bar{x}}_i$ values for the operators are different, it indicates the operator bias since all the operators are measuring the same part. The reproducibility can be calculated as

$$\hat{\sigma}_{reproducibility} = \frac{R_{\bar{\bar{x}}}}{d_2}$$

where, $R_{\bar{\bar{x}}}$ is calculated as shown below.

:

:

:

:

and d_2 = 1.693 for a sample size of three (there are three operators). Using these values, the value of d_2 can be obtained from the table. Using the value of d_2 :

$$\hat{\sigma}_{reproducibility} = \frac{R_{\bar{\bar{x}}}}{d_2} = \frac{0.62}{1.693} = 0.37$$

The measurement error can now be estimated using the repeatability and reproducibility values.

:

:

:

Therefore, $\hat{\sigma}_{gage} = \ldots.$

The gage capability using the P/T ratio,

$$\frac{P}{T} = \frac{6\hat{\sigma}}{(USL - LSL)} = \ldots = 0.101$$

Since the value of P/T = 0.101 (greater than 0.10), the gage is not adequate. Further operator training .....

Gage R&R Study

The purpose of measurement system analysis is to assess the variance components and determine how much of the variation is due to the measurements. The measurement system analysis is commonly known as Gage R&R Study. The variances to be analyzed are shown in Figure 7.5.

:

The total measurement variation involves two components; variation due to the product or part, and the variation due to measurement error or gage. This total variation can be written as:

$$\sigma^2_{total} = \sigma^2_{part} + \sigma^2_{gage} \quad (7.2)$$

Where,

$$\sigma^2_{measurementerror} = \sigma^2_{gage} = \sigma^2_{repeatability} + \sigma^2_{reproducibility} \quad (7.3)$$

The variance components explained in equations (2) and (3) are shown in Figure 7.5.The percentage variation due to the measurement system or %R&R is estimated as

$$\% R \ \& \ R = \frac{\sigma_{measurement}}{\sigma_{total}} X\,100\%$$

Methods of Gage Analysis

Figure 7.6 shows the methods of gage analysis. These methods are discussed with examples in subsequent sections.

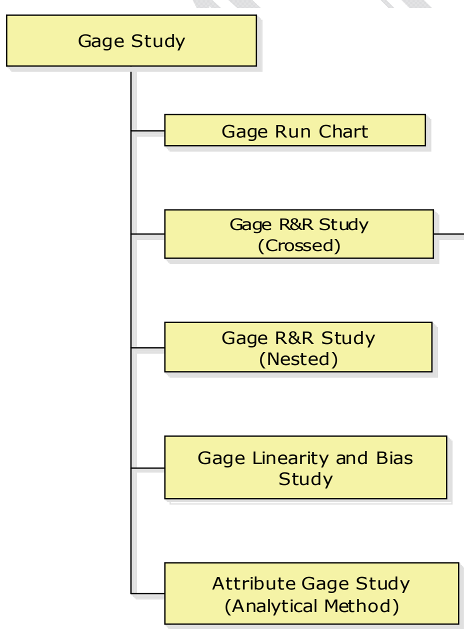

Figure 7.6: Methods of Gage R&R Analysis

Graphical Analysis of Gage Study: Gage Run Chart

The gage run chart is a graphical way of assessing the measurement errors. It provides a plot of the measured values by operator and part number. The plot is a simple way of looking into the variations in the measured values. The variation in measurements due to operators or parts can be seen from this plot.

Gage Run Chart: Example 1

Table 7.2 shows the measurements on a sample of eight parts selected from a manufacturing process. The parts represent the normal variation of the process. Three operators were selected to measure the parts. Each operator measured the eight parts with the same instrument three times in a random order. The measured values are shown below. We will use the gage run chart to assess the variation in measurements due to parts and operators.

Table 7.2

Operator A								
Trials	*Part 1*	*Part 2*	*Part 3*	*Part 4*	*Part 5*	*Part 6*	*Part 7*	*Part 8*
1	69	98	80	82	58	99	87	69
2	68	97	78	85	56	98	89	72
3	65	95	79	86	50	99	91	71
Operator B								
Trials	*Part 1*	*Part 2*	*Part 3*	*Part 4*	*Part 5*	*Part 6*	*Part 7*	*Part 8*
1	62	99	79	81	50	99	85	73
2	64	93	77	78	51	102	89	72
3	65	96	78	79	52	101	80	70

:

Table 7.2 must be entered as shown in **GAGE3.MTW** to do the plot. The steps to construct the plot are shown in Table 7.3.

Table 7.3

Gage Run Chart (1)	Open the worksheet **GAGE3.MTW** From the main menu, select **Stat ➢Quality Tools ➢Gage Study ➢Gage Run Chart** In the **Gage Run Chart** dialog box, select or type : Click the **Gage Info** tab and provide the details about the Gage : : Type-in a **Historical Mean** value (or the mean will be calculated from the data) Click **OK**

The gage run chart is shown in Figure 7.7.

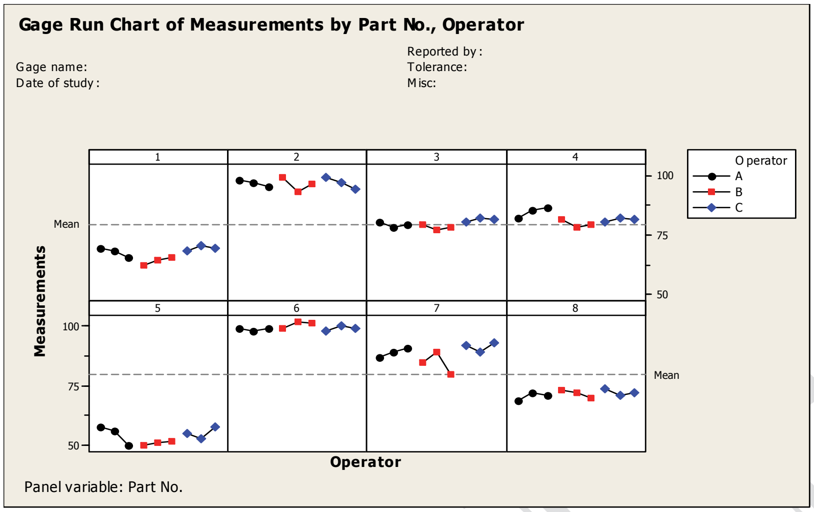

Figure 7.7: A Gage Run Chart of Measurements by Part Number and Operator of GAGE3.MTW

INTERPRETING THE RESULTS

(a) Figure 7.7 shows the measurement results for the eight parts by each of the three operators. Each operator measured the parts 3 times (3 trials). Each column in Figure 7.7 represents a part (1 through 8). Within each column, the measurements by 3 operators are represented by different symbols. The dotted lines represent the mean of the measured values. This mean line (or the reference line) helps to see

:

(b) Figure 7.7 shows that part-to-part variation is dominant. The measurements for parts 3 and 4 are close to the reference line. For all the other parts, the measurements are above or below the reference line. In fact, the measured values are far away from the reference line for all the parts except parts 3 and 4. If there is a significant variation from part-to-part,…………….

(c) The plot also provides an idea about repeatability and reproducibility, which are variations due to the gage or measurement system and operators. Recall that repeatability of a measuring instrument refers to how well the instrument is repeatedly able to measure the same characteristic under the same condition. The measurements by the three operators on each of the parts show that the measured values are close to each other……………………

(d) We can also check the reproducibility from the run chart in Figure 7.7. Reproducibility is the variation due to different operators using the same measuring instrument at different time periods or under different …..

(e) ***The conclusion from Figure 7.5 is that part-to-part variation is dominant*.**

Gage Run Chart: Example 2

The worksheet **GAGE2.MTW** shows the measured values for three selected parts. The parts are indicative of the range of process variation. Three operators were selected to measure the parts…………

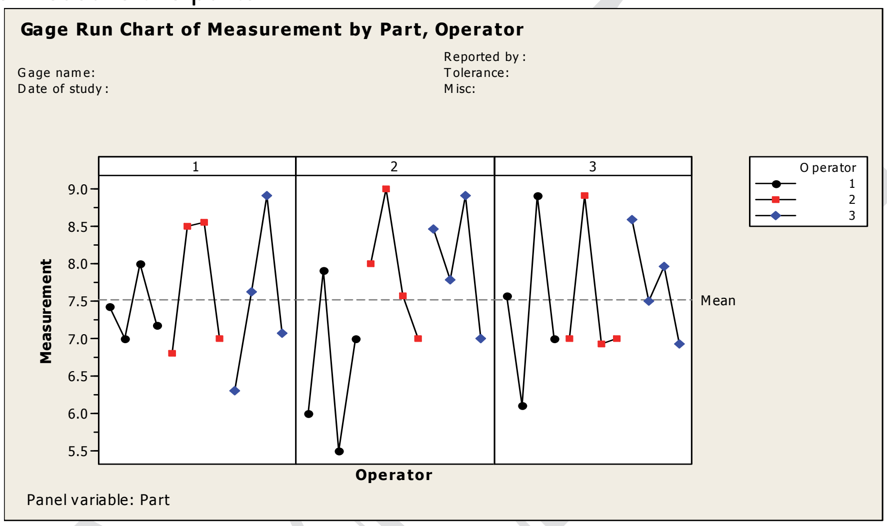

Figure 7.8: A Gage Run Chart of Measurements by Part Number and Operator of GAGE2.MTW

In this case, repeatability is the dominant factor.

Case 1: Determining Gage Capability

This example demonstrates how to assess gage capability when one operator takes multiple measurements on selected parts. Twenty parts are selected and the operator, who usually performs the measurements, measured each of the twenty selected parts twice. The measurements are shown in Table 7.8.

Table 7.8

Part	Operator A		$\bar{x}$	R
	Trial 1	Trial 2		
1	21	20	20.5	1
2	24	23	23.5	1
3	20	21	20.5	1
4	27	27	27.0	0
5	19	18	18.5	1
6	23	21	22.0	2
7	22	21	21.5	1
8	19	17	18.0	2
9	24	23	23.5	1
10	25	23	24.0	2
11 : :	21	20	20.5	1 Cont....
12	18	19	18.5	1
13	23	25	24.0	2
14				0
15				1
16				0
17				0
18				2
19				1
20	19	19	19.0	0
			$\bar{\bar{x}} = 22.3$	$\bar{R} = 1.0$

Note that there is only one operator involved in the measurement process so we can only determine the repeatability and part-to-part variation. Using the data in Table 7.8, the data file **GAGE1A.MTW** was created for analysis.

1. Analysis using Gage Run Chart

We will first create a gage run chart of data in Table 7.8. It is a quick way to see different variance components. For example, from this chart we can see which variance component is dominant: part-to-part variation, or measurement variation. To construct a gage run chart, follow the instructions in Table 7.9.

Table 7.9

GAGE RUN CHART	Open the worksheet **GAGE1A.MTW** From the main menu select, **Stat** ➢ **Quality Tools** ➢ **Gage Study** ➢ **Gage Run Chart** In the **Gage Run Chart** dialog box, select or type : : Click **OK**

The gage run chart is shown in Figure 7. 12.

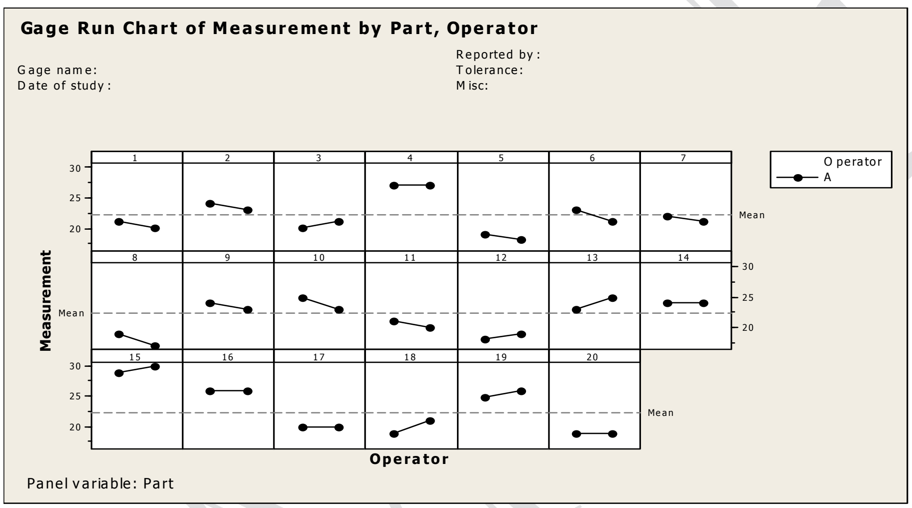

Figure 7.12: Gage Run Chart for the Data of GAGE1A.MTW

INTERPRETING THE RESULTS

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- The run chart shows that part-to-part variation is dominant. The measurements for a few parts are close to the reference line. For the majority of parts, the measurements are above or below the reference line, indicating part-to part variation. We can also see some type of pattern. If there is a significant variation from part-to-part, some type of pattern will appear (measured values being up or down).

- The plot in Figure 7.12 also provides an idea about repeatability (which is the variation due to the gage or measurement system)……………….

2. ANALYSIS USING GAGE R&R (CROSSED): ANOVA METHOD OF THE DATA IN GAGE1A.MTW

Here, we used the Gage R&R (crossed): ANOVA method in MINITAB to get a quantitative analysis. Gage R&R (crossed) method in MINITAB provides two options: (1) the X-bar and R method, and (2) ANOVA method. When there is only one operator (as in this case), the X-bar and R method ………….The results are displayed on the session window and the graphs are shown separately on the graphics window.

Table 7.10

GAGE R&R STUDY (CROSSED)	Open the worksheet **GAGE1A.MTW** From the main menu, select **Stat ➢Quality Tools ➢Gage Study ➢Gage R&R Study (Crossed)** : ; click **OK** Click the **Options** tab In the **Study Variation** box, type **5.15** Under the **Process tolerance** , type **55 in the box Upper spec-Lower spec** Click **OK** Under Method of Analysis, click the circle next to **ANOVA** Click **OK** in all dialog boxes

Table 7.11 shows the results from the ANOVA method. The plots are shown in Figure 7.13.

Table 7.11

```
Gage R&R Study - ANOVA Method
* NOTE * No or identical values for Operator - will analyze data without
         operator factor.
One-Way ANOVA Table 7.
Source         DF     SS        MS         F       P
Part           19   377.4   19.8632   26.4842   0.000
Repeatability  20    15.0    0.7500
Total          39   392.4
Gage R&R
                                    %Contribution
Source            VarComp          (of VarComp)

Total Gage R&R     0.7500               7.28
  Repeatability    0.7500               7.28
Part-To-Part       9.5566              92.72
Total Variation   10.3066             100.00
                                   Study Var  %Study Var  %Tolerance
Source            StdDev (SD)      (6 * SD)       (%SV)   (SV/Toler)

Total Gage R&R        0.86603       5.1962       26.98         9.45
  Repeatability       0.86603       5.1962       26.98         9.45
Part-To-Part          3.09137      18.5482       96.29        33.72
Total Variation       3.21039      19.2623      100.00        35.02
Number of Distinct Categories = 5
```

INTERPRETING THE RESULTS IN TABLE 7.11

When only one operator measures the parts or the operator is not entered in the data file, a one-factor ANOVA model is fitted. The One-way ANOVA table in Table 7.11 shows that the variation due to part is significant (p-value for part is 0.000).

The percent contribution of variance component (%Contribution) column under Gage R&R shows that 7.28% of the variation is due to Gage R&R. All this variation is due to repeatability or measuring equipment. There is only one operator involved, so the reproducibility part is missing. ***The % Contribution of Part-to-Part is 92.72%. It is clear that part-to-part variation is dominant in this case.*** The variation due to measuring equipment (7.28%) is small-indicating that the gage is capable. Further analysis will tell more about the gage capability. Further analyses of Table 7.11 are explained below.

From Gage R&R analysis, the variance due to repeatability or gage is 0.7500. This is reported as **MS Repeatability** in the One-Way ANOVA table.

$$\sigma^2_{repeatability} = 0.7500$$

Since there is no variation due to the operator in this example, the reproducibility part is missing from the analysis. The standard deviation of measurement error is σ_{gauge}. This value is reported in **Total Gage R&R** row and **StdDev (SD)** column and can be written as

$$\sigma_{gauge} = 0.86603$$

This value is the square root of 0.75000, and is reported under the **StdDev (SD)** column of the **Total GAGE R&R** row.

The **gage capability** is given by $6\sigma_{gauge}$ assuming that the measurement error is normally distributed. The value of gage capability is reported under the **Study Var (6*SD)** column and **Total Gage R&R row**. The value is calculated as shown below.

$$6\sigma_{gauge} = 6(0.86603) = 5.1962$$

The Gage capability means that the individual measurements are expected to vary as much as

$$\pm 3\sigma_{gauge} = \pm 3(0.86603) = \pm 2.598$$

due to gage error. **Percent Tolerance (% Tolerance):** The percent tolerance or precision-to-tolerance is calculated as

$$\%Tolerance = \frac{6\sigma_{gauge}}{USL - LSL} = \frac{SV}{Tolerance} = \frac{5.1962}{60-5} = 0.0945 or 9.45\%$$

This value is reported under the **%Tolerance** column and **Total Gage R&R** ……………….

The measurement may also be expressed in the following way:

$$\frac{\sigma_{gauge}}{\sigma_{part}} X100 = \frac{0.86603}{3.09137} X100 = 28.01\%$$

The values are from Table 7.11 under the **StdDev (SD)** column. This ratio does not require the tolerance value.

Reminder: The formulas to estimate the variance and standard deviation (part-to-part and repeatability) can be obtained from the help screen of GAGE R&R (crossed): ANOVA method in MINITAB.

Analysis of Graphs

Several graphs are produced as a part of the analysis. The graphs from this analysis are shown in Figure 7.13. The interpretation for each graph is provided below.

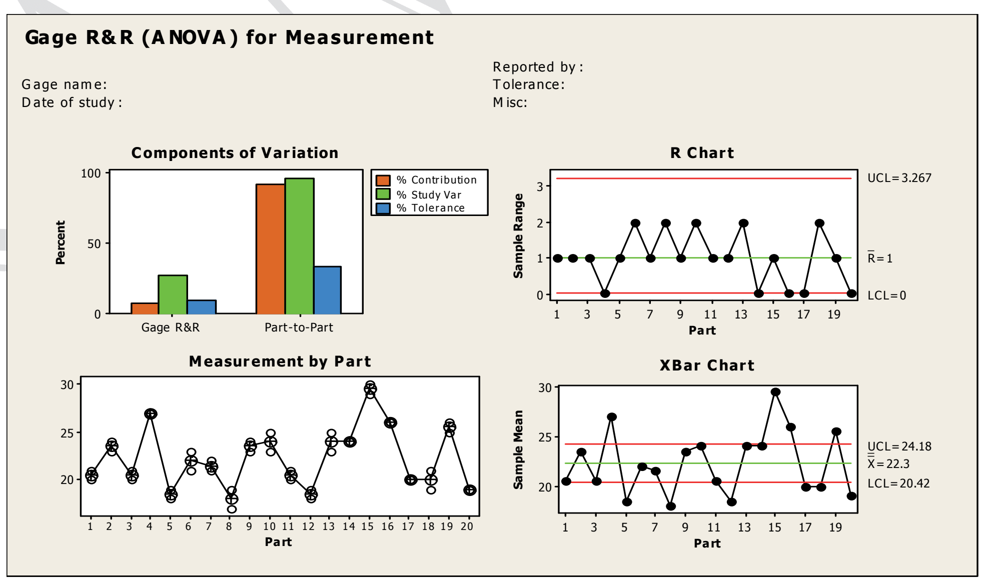

Figure 7.13: Plots for Gage R&R Analysis: ANOVA Method

The *component of variation* graph shows that part-to-part variation is dominant. The Gage R&R variation is much smaller than the part-to-part variation. Gage R&R

variation is the variation due to repeatability and reproducibility. In this case, the reproducibility or variation due to operators is missing,

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Measurement by Part Graph

This plot in Figure 7.13 shows a clear part-to-part variation in measurements. The average of the measurements for each part is connected using a straight line.

X-Bar Chart

chart shows many out of control points. *When part-to-part variation is dominant, an Xbar chart will have out-of-control points*. The chart shows the ability of the gage (or measuring equipment) to ……………….

R-Chart

The R chart shows the gage capability or the magnitude of measurement error. The points on the R chart show the difference between measurements on the same part using the same measuring equipment. *The R chart in our example is within control, which means that the operator is not having any problem in making consistent measurements. When the R chart shows out of control points, it is an indication that the operator is having difficulty using the equipment.*

Case 2: Determining Gage Capability

In this example, twenty parts are measured by 3 operators. Each operator measures the twenty parts twice or, three operators take multiple measurements of the selected parts. The variation in this case would be part-to-part variation, the variation due to measurement instrument or repeatability, and variation due to operators or reproducibility. This case has three components: repeatability, reproducibility, and part-to-part variation.

The measurements are shown in Table 7.12. Using the data in Table 7.12 the worksheet **GAGE1B.MTW** was created. We analyzed the data using the following methods in MINITAB:

1. Gage Run Chart
2. Gage R&R (Crossed): Xbar and R Method
3. Gage R&R (Crossed): ANOVA Method

The ANOVA method is more accurate because it takes into account the operator and operator-by-part interaction.

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1. *Analysis using Gage Run Chart*

Using the worksheet **GAGE1B.MTW**, we created the gage run chart shown in Figure 7.14. To do this chart, open the worksheet **GAGE1B.MTW** and follow the instructions in Table 7.9.

Table 7.12

Part	Operator A		Operator B		Operator C	
Trail 2	Trail 1		Trail 1	Trial 2	Trail 1	Trial2
1	51	56	50	50	49	51
2	54	57	54	54	53	54
3	49	55	49	51	50	52
4	57	63	58	56	57	58
5	49	42	49	48	48	51
6	53	58	54	51	53	52
7	52	58	52	54	52	50

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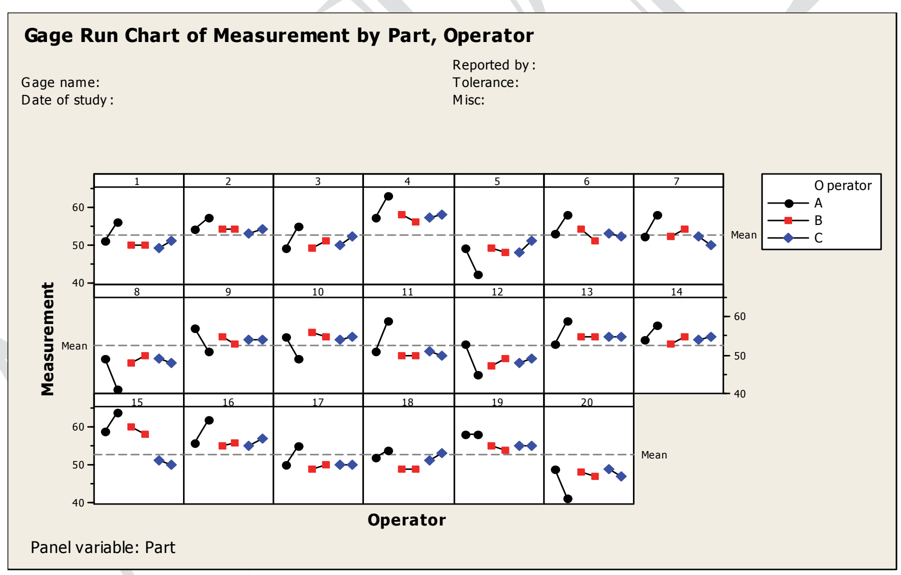

Figure 7.14: Gage Run Chart for the Measurement Data in GAGE1B.MTW

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2. Analysis using Gage R&R Study (Crossed): X-bar and R Method

We analyzed the data in file **GAGE1B.MTW** using the X-bar and R method**.** A gage run chart of this data is shown in Figure 7.14. While the gage run chart is a graphical way of looking into the variations in the measured data, the X-bar and R chart provides a quantitative analysis. We will compare the conclusions from Figure 7.14 with the X-bar and R method in this section. Follow the steps in Table 7.13 for the X-bar and R method.

Table 7.13

GAGE R&R (CROSSED) Study	Open the worksheet **GAGE1B.MTW** From the main menu select, **Stat ➢Quality Tools ➢Gage ➢Gage R&R Study (Crossed)**
: :	Under **Method of Analysis**, click the circle next to **Xbar and R** Click **OK**

The results shown in Table 7.14 are displayed on the session window. The plots are shown in Figure 7.15.

Table 7.14

Gage R&R Study - XBar/R Method

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	6.0397	39.61
Repeatability	5.5192	36.20
Reproducibility	0.5206	3.41
Part-To-Part	9.2073	60.39
Total Variation	15.2471	100.00

Source	StdDev (SD)	Study Var (6 * SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	2.45759	14.7455	62.94	19.66
Repeatability	2.34929	14.0957	60.16	18.79
Reproducibility	0.72151	4.3291	18.48	5.77
Part-To-Part	3.03436	18.2062	77.71	24.27
Total Variation	3.90475	23.4285	100.00	31.24

Number of Distinct Categories = 1

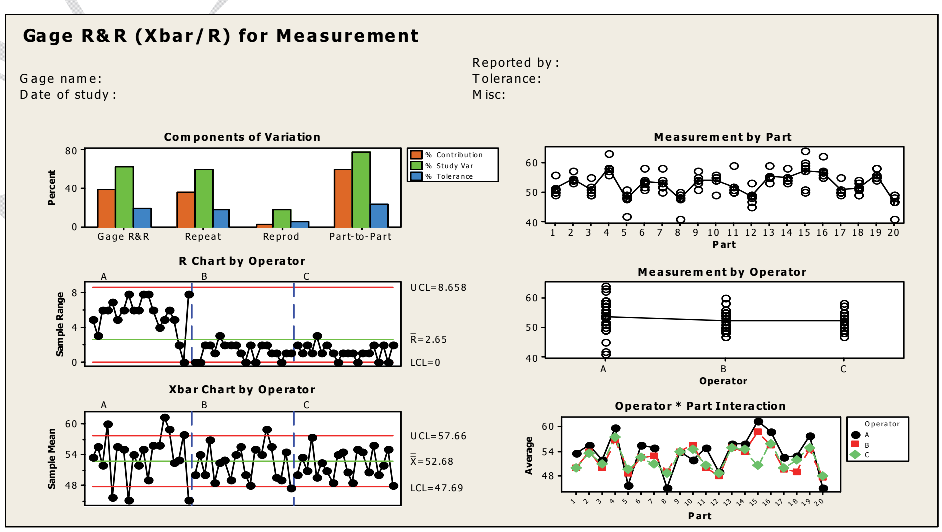


Figure 7.15: Plots using Gage R&R (Crossed): XBar and R Method of Data in

GAGE1B.MTW

Table 7.14 provides the variance components, and the percentage of the variance components relative to the total variance…………

INTERPRETATION OF PLOTS IN FIGURE 7.15

The component of variation plot shows the percentage of variation due to Gage R&R which is sums of the variations due to repeatability, reproducibility, and the percent variation of part-to-part. Each component of variation– gage R&R, repeatability, reproducibility, and part-to-part–has three bars. The first bar shows …..

3. ANALYSIS USING GAGE R&R STUDY (CROSSED): ANOVA METHOD

Table 7.15

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table 7. With Interaction

Source	DF	SS	MS	F	P
Part	19	1249.83	65.7803	8.57954	0.000
Operator	2	57.65	28.8250	3.75957	0.032
Part * Operator	38	291.35	7.6671	1.18716	0.272
Repeatability	60	387.50	6.4583		
Total	119	1986.33			

Two-Way ANOVA Table 7. Without Interaction

Source	DF	SS	MS	F	P
Part	19	1249.83	65.7803	9.49616	0.000
Operator	2	57.65	28.8250	4.16123	0.018
Repeatability	98	678.85	6.9270		
Total	119	1986.33			

Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	7.4745	43.25
Repeatability	6.9270	40.08
Reproducibility	0.5474	3.17
Operator	0.5474	3.17
Part-To-Part	9.8089	56.75
Total Variation	17.2834	100.00

Source	StdDev (SD)	Study Var (6 * SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	2.73395	16.4037	65.76	29.82
Repeatability	2.63193	15.7916	63.31	28.71
Reproducibility	0.73990	4.4394	17.80	8.07
Operator	0.73990	4.4394	17.80	8.07
Part-To-Part	3.13191	18.7915	75.33	34.17
Total Variation	4.15733	24.9440	100.00	45.35

Number of Distinct Categories = 1

INTERPRETATION OF GRAPHICAL OUTPUTS OF GAGE R&R: XBAR AND R STUDY

The Gage R&R: Xbar and R analysis provides graphs for the analysis of measurement data. Figure 7.16 shows the graphs for this example.

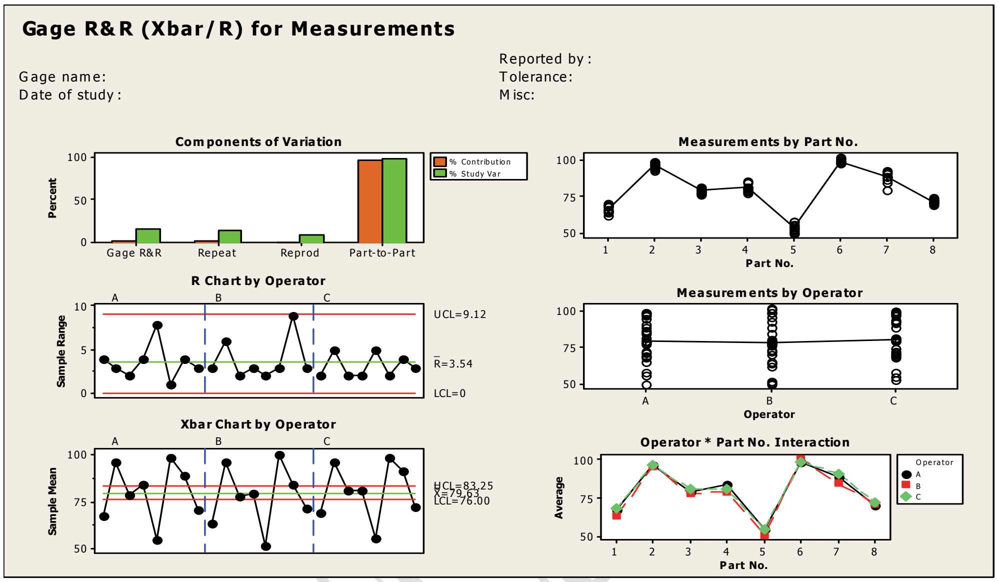


Figure 7.16: Graphs for Gage R&R: Xbar and R Method GAGE3.MTW Data

INTERPRETATION OF PLOTS IN FIGURE 7.16

The Gage R&R study produces six graphs shown in Figure 7.16. These graphs provide additional insight for improvement opportunities. Each graph in Figure 7.16 is described below.

Components of Variation

This graph provides bars for each of the variance components including Gage R&R, repeatability, reproducibility, and part-to-part variation. Note that the graph does not provide the bar for operator or operator by part variance.

:

:

Other Examples in the Chapter

Case 3: Gage R&R Study (Crossed): Xbar and R Method

Case 4: Gage R&R Study (Crossed): ANOVA Method using Case 3 Data

Case 5: Comparing the Results of The Gage Run Chart, The Gage R&R: Xbar and R Method, and The Gage R&R: ANOVA Method

The data file **GAGE2.MTW** shows the measurements of three selected parts. The parts are indicative of the range of process variation (full scale). Three operators were selected to measure the parts. Each operator measured the parts four times in a random order. The measured values are shown in Table 7.4. The worksheet **GAGE2.MTW** was created using the data in Table 7.4. Using the worksheet **GAGE2.MTW**:

- Create a Gage Run Chart
- Perform a Gage R&R Study (Crossed) using the Xbar and R Method
- Perform a Gage R&R Study (Crossed) using the ANOVA Method

Compare the results from the above methods.

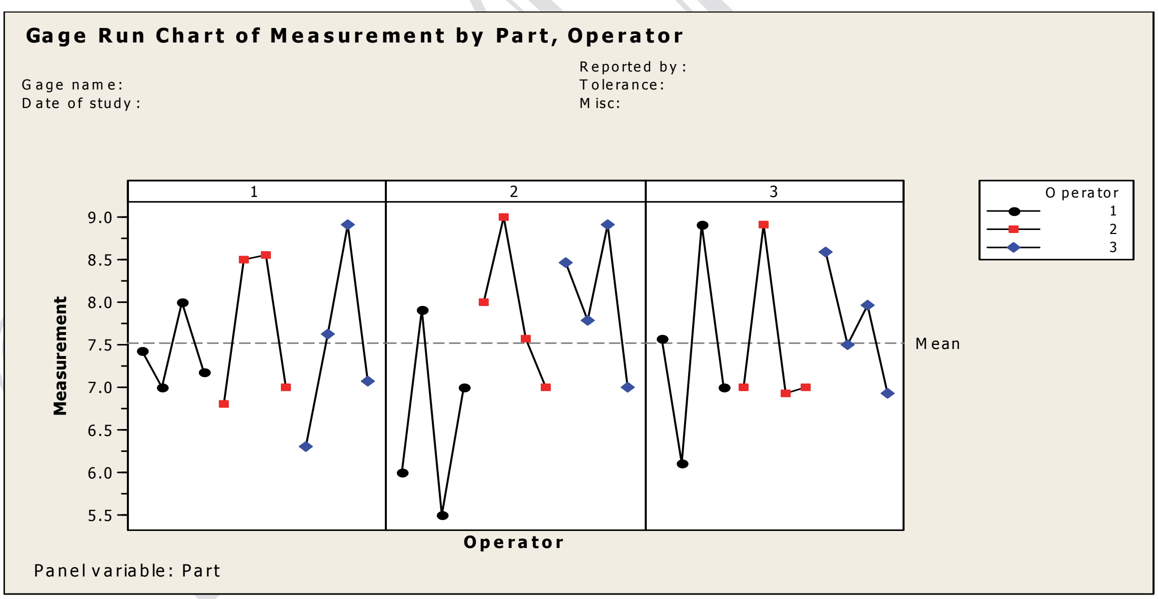


Figure 7.19: Gage Run Chart of Data in GAGE2.MTW

Case 6: Another Example on Comparing the Results of a Gage Run Chart, Gage R&R: Xbar and R Method, and A Gage R&R: ANOVA Method

The data file **GAGE4.MTW** shows the measured values on a sample of 10 parts from a manufacturing process. A gage run chart for this data was shown in Figure 7.10. An investigation of Figure 7.10 shows repeatability, reproducibility, and part-to-part to variations in the data. Here we will analyze the data using the Gage R&R: Xbar and R, and Gage R&R: ANOVA methods.

Gage R&R: Xbar and R Method:

Open the work sheet **GAGE4.MTW**

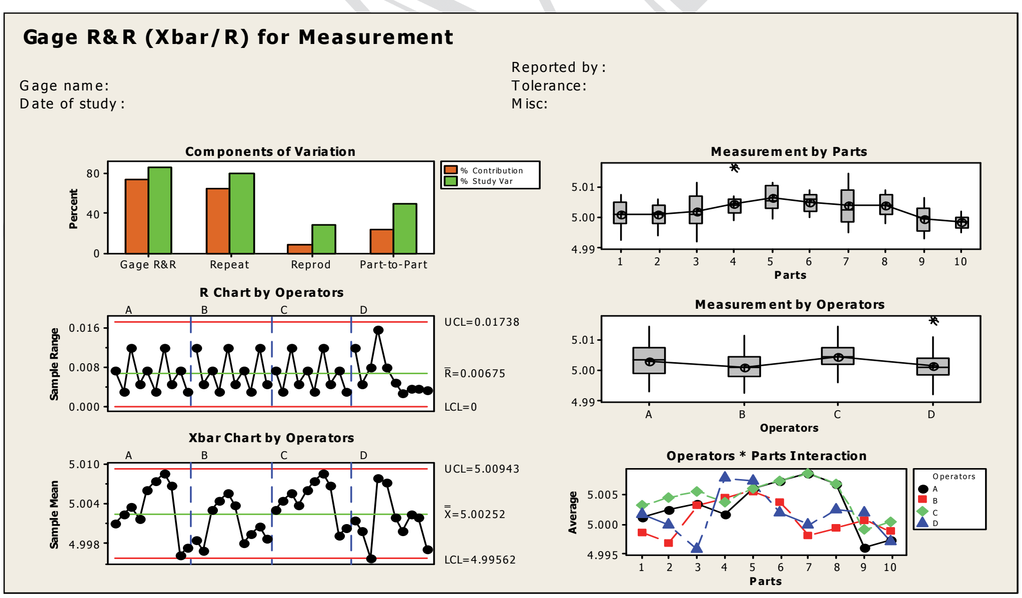
- To perform a Gage R&R Study (Crossed) using Xbar and R Method, follow the steps in Table 7.10. Make sure you select the appropriate variable names in the dialog boxes. The results of the Xbar and R method are shown in Figure 7.22.

Gage R&R Study - XBar/R Method

```
                                 %Contribution
Source                VarComp    (of VarComp)

Total Gage R&R      0.0000180           74.80
  Repeatability     0.0000159           66.15
  Reproducibility   0.0000021            8.64
Part-To-Part        0.0000061           25.20
Total Variation     0.0000240          100.00


                                 Study Var  %Study Var
Source              StdDev (SD)   (6 * SD)       (%SV)
Total Gage R&R        0.0042398  0.0254388       86.49
  Repeatability       0.0039873  0.0239236       81.33
  Reproducibility     0.0014414  0.0086482       29.40
Part-To-Part          0.0024611  0.0147665       50.20
Total Variation       0.0049023  0.0294140      100.00
```



Number of Distinct Categories = 1

Figure 7.22: Gage R&R: X bar/R Method for GAGE4.MTW Data in Example 8

Case 7: Gage R&R Study (Nested): ANOVA Method

The Gage R&R Study (Nested) uses the ANOVA method of analysis. This method is used when only one operator measures each part. The basic assumption is that all parts within a single batch are identical so that we can claim that the parts are the same. This is unlike the Gage R&R Study (Crossed) where the same part could be measured by multiple operators.

In the Gage R&R Study (Nested), the part is nested within the operator because each operator measures unique parts, therefore the data are analyzed using a nested design.

Determining the Bias and Linearity

Besides repeatability and reproducibility, the other part of a measurement system analysis is to determine the accuracy or bias and linearity. *Accuracy is defined as the difference between the measured value and the part's actual value or the master value*. The accuracy is divided into following three components

(a) Linearity
(b) Bias, and
(c) Stability.

Gage linearity is the measure of accuracy or bias of the measurements through the expected range of the measurements. The linearity determines if the gage has the same accuracy for different sizes of parts being measured. It also tells us how the size of the part affects the accuracy of the measurement system.

The bias (or gage accuracy) determines the difference between the observed average measurement and the master or true value.

The stability or the drift is the total variation in measurements when the measurements are obtained with the same measurement equipment on the same part while measuring a single characteristic over an extended period of time.

Case 8: Gage Linearity and Accuracy (Bias) Study

Example 1: To determine the linearity and bias of a gage, five parts were selected from a manufacturing process. These parts represent the entire operating range (or full scale) of the measurements. Each of the selected five parts was measured by the tool room to determine the master value. Once the master values for the parts were determined, an operator measured each of the parts 15 times randomly. The data are shown in Table 7.22. Using this data, the data file **GAGELIN1B.MTW** was created. Use this data file to

(a) determine the process variation using Gage R&R Study: ANOVA Method, and
(b) use the process variation to determine the gage linearity and bias.

Note: A Gage Linearity and Bias Study can also be conducted without knowing the process variation.

Comparing Two Measuring Instruments for Precision and Accuracy

Comparing two measuring instruments might be necessary in cases when the vendor's and customer's measurements are not consistent, or do not agree. If both parties are using the same measuring instruments and the same measuring procedure, simple tests can be performed to compare the accuracy and precision of the instruments. In this section we will be discussing two tests involving ***variables measurement.***

(1) comparing variances (precision) for paired data
(2) comparing the average of measurements obtained by two measuring instruments using a paired t-test

The first test determines if the two instruments have the same precision as measured by the standard deviation of the measurements. The second test determines if there is a significant difference in the average measurements of the two instruments. In other words, the first test determines the precision while the second test deals with the accuracy.

Case12: Use of Individuals Control Chart to Detect the Shift in Measuring Instruments

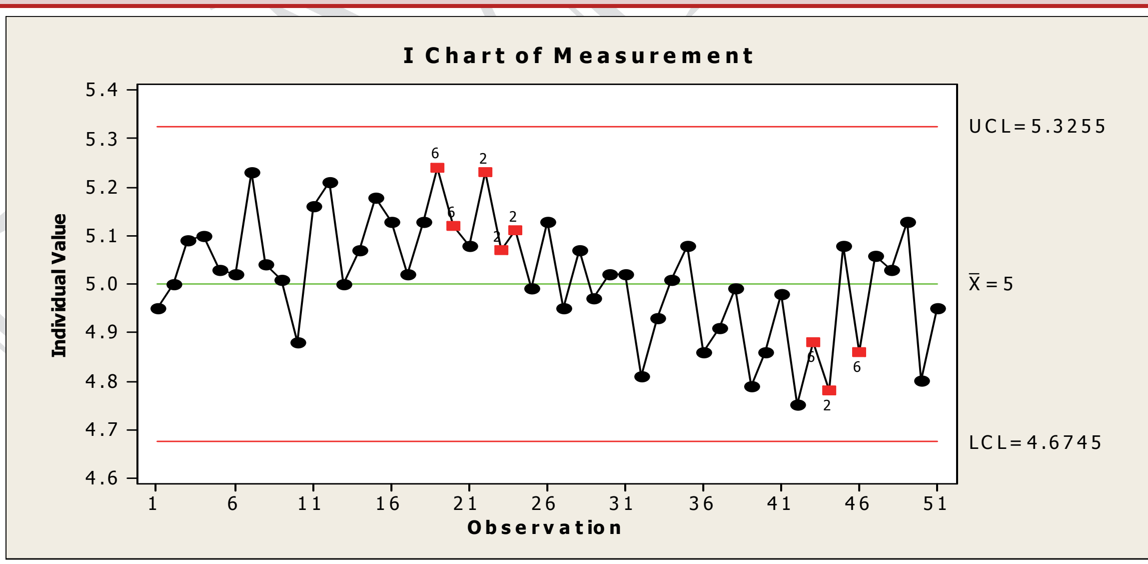


Figure 7.30: Individuals Chart of Measurement

Figure 7.30 shows no out of control points but the test results for special causes in Table 7. 36 shows the following

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 22, 23, 24, 44

The rule of "9 points on the same side of the center line" is violated on days 22, 23, 24 and again on day 44. The chart in Figure 7.30 shows a pattern where the measured points are plotting above the centerline until about day 26, and then start to plot below the centerline. This type of up and down pattern is indicative of a shift or drift from the nominal value of 5.0 in. The pattern in the individuals chart shows a shift in the upward direction and eventually a downward shift.

Chapter 6 of Six Sigma Volume 1 contains detailed analysis and interpretation of process capability analysis with data files and step-wise computer instructions for both normal and non-normal data.