Six Sigma Quality: Concepts & Cases- Volume I

STATISTICAL TOOLS IN SIX SIGMA DMAIC PROCESS WITH MINITAB® APPLICATIONS

Chapter 6

PROCESS CAPABILITY ANALYSIS FOR SIX SIGMA

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CHAPTER HIGHLIGHTS

This chapter deals with the concepts and applications of process capability analysis in Six Sigma. Process Capability Analysis is an important part of an overall quality improvement program. Here we discuss the following topics relating to process capability and Six Sigma:

1. *Process capability concepts and fundamentals*
2. *Connection between the process capability and Six Sigma*
3. *Specification limits and process capability indices*
4. *Short-term and long-term variability in the process and how they relate to process capability*
5. *Calculating the **short-term** or **long-term process capability***
6. *Using the process capability analysis to:*
    - *assess the process variability*
    - *establish specification limits (or, setting up realistic tolerances)*
    - *determine how well the process will hold the tolerances (the difference between specifications)*
    - *determine the process variability relative to the specifications*
    - *reduce or eliminate the variability to a great extent*
7. *Use the process capability to answer the following questions:*
    - *Is the process meeting customer specifications?*
    - *How will the process perform in the future?*
    - *Are improvements needed in the process?*
    - *Have we sustained these improvements, or has the process regressed to its previous unimproved state?*
8. *Calculating process capability reports for normal and non-normal data using MINITAB.*

CHAPTER OUTLINE

This document contains explanation and examples on process capability analysis from Chapter 6 of our Six Sigma Volume 1. The book contains numerous cases, examples and step-wise computer instruction with data files.

PROCESS CAPABILITY

Process Capability is the ability of the process to meet specifications. The capability analysis determines how the product specifications compare with the inherent variability in a process. The inherent variability of the process is the part of process variation due to common causes. The other type of process variability is due to the special causes of variation.

It is a common practice to take the six-sigma spread of a process's inherent variation as a measure of process capability when the process is stable. Thus, the process spread is the process capability, which is equal to six sigma.

PROCESS CAPABILITY ANALYSIS: AN IMPORTANT PART OF AN OVERALL QUALITY IMPROVEMENT PROGRAM

The purpose of the process capability analysis involves assessing and quantifying variability before and after the product is released for production, analyzing the variability relative to product specifications, and improving the product design and manufacturing process to reduce the variability. Variation reduction is the key to product improvement and product consistency.

The process capability analysis is useful in determining how well the process will hold the tolerances (the difference between specifications). The analysis can also be useful in selecting or modifying the process during product design and development, selecting the process requirements for machines and equipment, and above all, reducing the variability in production processes.

DETERMINING PROCESS CAPABILITY

The following points should be noted before conducting a process capability analysis.

- Process capability should be assessed once the process has attained statistical control. This means that the special causes of variation have been identified and eliminated. Once the process is stable, ………….
- In calculating process capability, the specification limits are required in most cases, …… Unrealistic or inaccurate specification limits may not provide correct process capability.

- Process capability analysis using a histogram or a control chart is based on the assumption that the process characteristics follow a normal distribution. While the assumption of normality holds in many situations, there are cases where the processes do not follow a normal distribution. Extreme care should be exercised where normality does not hold….

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SHORT-TERM AND LONG-TERM VARIATION

The standard deviation that describes the process variation is an integral part of process capability analysis. In general, the standard deviation is not known and must be estimated from the process data. The estimated standard deviation used in process capability calculations may address "short-term" or "long-term" variability. The variability due to common causes is described as "short-term" variability, while the variability due to special causes is considered "long-term" variability.

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Some examples of "long-term" variability may be lot-to-lot variation, operator-to-operator variation, day-to-day variation or shift-to-shift variation. Short-term variability may be within-part variation, part-to-part variation, variations within a machine, etc. However, the literature differs on what is "long-term" and what is "short-term" variation.

In process capability analysis, both "short-term" and "long-term" indexes are calculated and are not considered separately in assessing process capability. The indexes Cp and Cpk are "short-term" capability indexes and are calculated using "short-term" standard deviation whereas, Pp and Ppk are "long-term" capability and are calculated using "long-term" standard deviation estimate. These are discussed in more detail later.

DETERMINING PROCESS CAPABILITY

Following are some of the methods used to determine the process capability.
The first two are very common and are described below.

(1) Histograms and probability plots,

(2) Control charts, and

(3) Design of experiments.

PROCESS CAPABILITY USING HISTOGRAMS: SPECIFICATION LIMITS KNOWN

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Suppose that the specification limits on the length is 6.00±0.05. We now want to determine the percentage of the parts outside of the specification limits. Since the measurements are very close to normal, we can use the normal distribution to calculate the nonconforming percentage. Figure 6.2 shows the histogram of the length data with the target value and specifications limits. To do this plot, follow the instructions in Table 6.2.

Table 6.2

HISTOGRAM WITH SPECIFICATION LIMITS	Open the worksheet **PCA1.MTW** From the main menu, select **Graph ➢ Histogram** Click on **With Fit** then click **OK** For **Graph variables, ………….**Click the **Scale** then click the **Reference Lines** tab In the Show **reference lines at data values** type **5.95 6.0 6.05** Click **OK** in all dialog boxes.

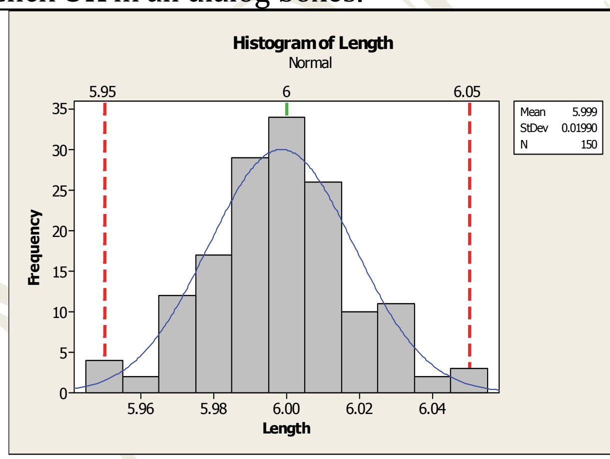

Figure 6.2: Histogram of the Length Data with Specification Limits and Target

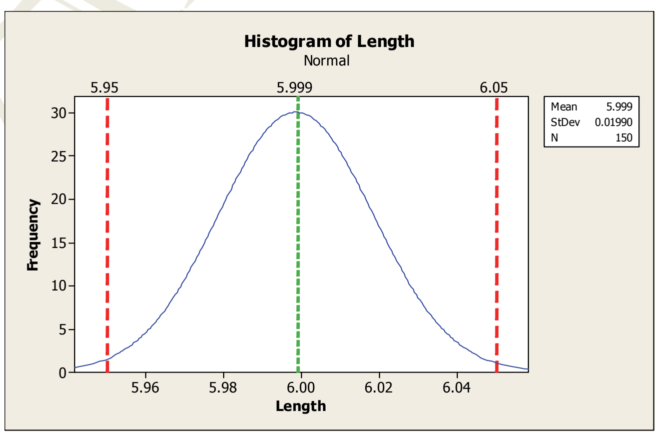

Figure 6.3: Fitted Normal Curve with Reference Line for the Length Data

Table 6.4

Cumulative Distribution Function
Normal with mean = 5.999 and standard deviation = 0.0199
x P(X <= x)
5.95 0.0069022
6.05 0.994809

From the above table, the percent conforming can be calculated as 0.994809 - 0.0069022 = 0.98790 or, 98.79%. Therefore, the percent outside of the specification limits is 1-0.98790 or, 0.0121 (1.21%).

Process Capability using Probability Plots

A probability plot can be used in place of a histogram to determine the process capability. Recall that a probability plot can be used to determine the distribution and shape of the data. If the probability plot indicates that the distribution is normal, the mean and standard deviation can be estimated from the plot. For the length data discussed above, we know that the distribution is normal. We will construct a probability plot (or perform a Normality test) …….

Table 6.5

NORMALITY TEST USING PROBABILITY PLOT	Open the worksheet **PCA1.MTW** From the main menu, select **Stat** ➢ **Basic Statistics** ➢**Normality Test** For **Variable**, Select **C1 Length** Under **Percentile Line**, …..type **5 0 84** in the box Under **Test for Normality**, ….**Anderson Darling** Click **OK**

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For a normal distribution, the mean equals median, which is 50th percentile, and the standard deviation is the difference between the 84th and 50th percentile. From Figure 6.4, the estimated mean is 5.9985 or 5.999 and the estimated standard deviation is

$\hat{\sigma} =$ 84th percentile - 50th percentile = 6.0183 - 5.9985 = 0.0198

Note that the estimated standard deviation is very close to what we got from earlier analysis. The process capability can now be determined as explained in the previous example.

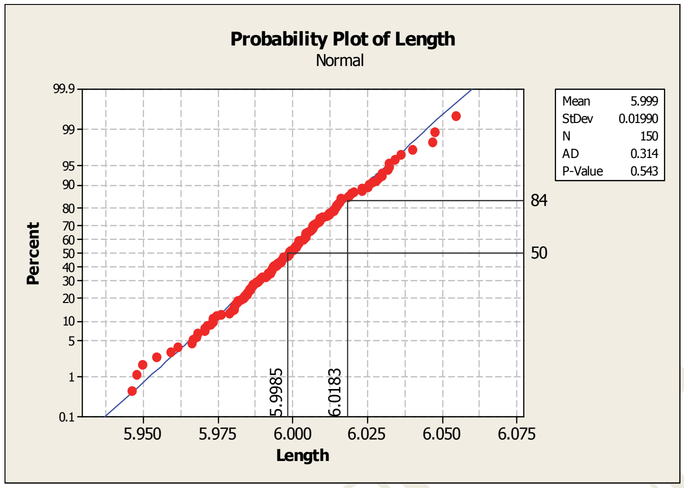


Figure 6.4: Probability Plot for the Length Data

Continued…

ESTIMATING PERCENTAGE NONCONFORMING FOR NON-NORMAL DATA: EXAMPLE 1

When the data are not normal, an appropriate distribution should be fitted to the data before calculating the nonconformance rate.

Data file **PCA1.MTW** shows the life of a certain type of light bulb. The histogram of the data is shown in Figure 6.5. To construct this histogram, follow the steps in Table 6. 1.

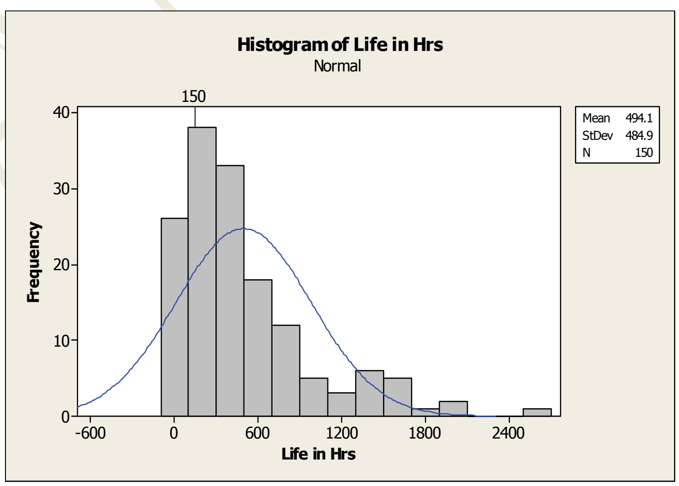


Figure 6.5: Life of the Light bulb

For the data in Figure 6.5, it is required to calculate the capability with a lower limit because the company making the bulbs wants to know the minimum survival rate. They want to determine the percentage of the bulbs surviving 150 hours or less.

The plot in Figure 6.5 clearly indicates that the data are not normal. Therefore, if we use the normal distribution to calculate the nonconformance rate, it will lead to a wrong conclusion. It seems reasonable to assume that the life data might follow an exponential distribution. We will fit an exponential distribution to the data ....

Table 6.6

FITTING DISTRIBUTION (EXPONENTIAL)	Open the worksheet **PCA1.MTW** From the main menu, select **Graph** ➢ **Histogram** Click on **With Fit** then click **OK** **:** Click the **Distribution** tab, check the box next to **Fit Distribution** Click the downward arrow and select **Exponential** Click **OK** in all dialog boxes.

The histogram with a fitted exponential curve shown in Figure 6.6 will be displayed. The exponential distribution seems to provide a good fit to the data. The parameter of the exponential distribution (mean =494.1 hrs) is ....

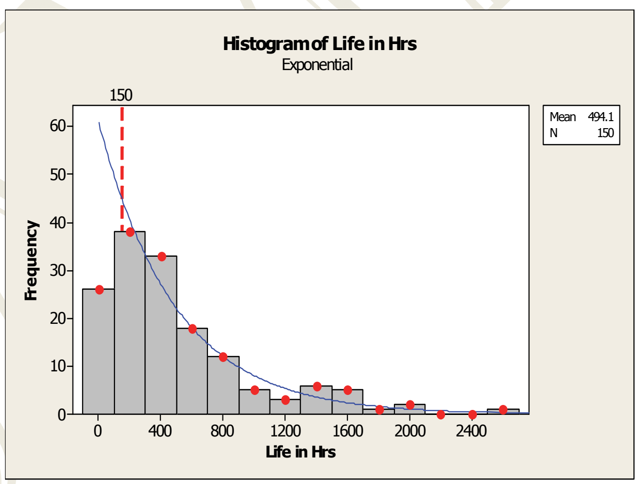
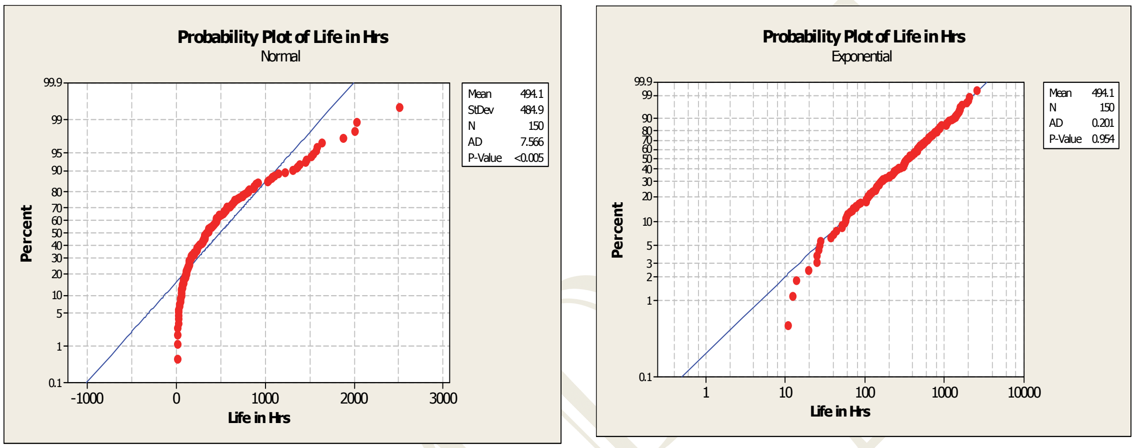


Figure 6.6: Exponential Distribution Fitted to the Life Data

The plot using the exponential distribution clearly fits the data as most of the plotted plots are along the straight line. To do the probability plots in Figure 6.7, follow the instructions in Table 6.7.........

Table 6.7

PROBABILITY PLOT	Open the worksheet **PCA1.MTW** From the main menu, select **Graph** ➢ **Probability Plots** Click on **Single** then click **OK** For **Graph variables**, type **C2** or select **Life in Hrs** Click on **Distribution** then select **Normal** ....Click **OK** in all dialog boxes.



(a) (b)

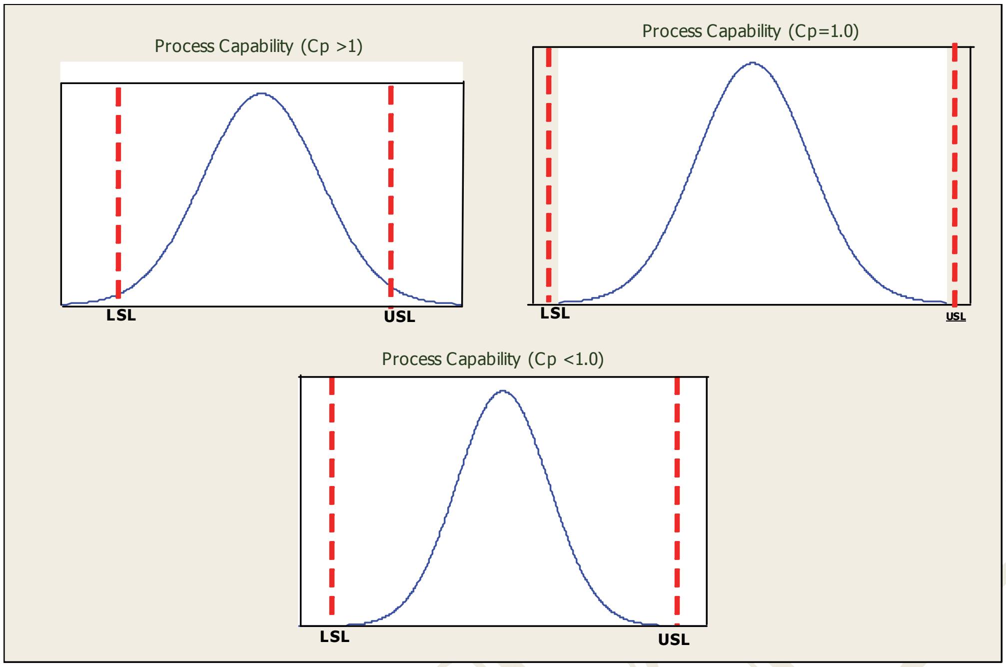
Figure 6.7: Probability Plots of the Life Data

Table 6.9

Cumulative Distribution Function	
Exponential with mean = 494.1	
x	P(X <= x)
150	0.261831

The table shows the calculated probability, P (x <= 150) = 0.2618. This means that 26.18% of the products will fail within 150 hours or less.

CAPABILITY INDICES FOR NORMALLY DISTRIBUTED PROCESS DATA



MINITAB provides several options for determining the process capability. The options can be selected by using the command sequence **Stat ➢Quality Tools ➢Capability Analysis**. This provides several options for performing process capability analysis including the following:

- Normal
- Between/Within
- Non-normal
- Multiple Variables (Normal)
- Multiple Variables (Nonnormal)
- Binomial
- Poisson

DETERMINING PROCESS CAPABILITIES USING NORMAL DISTRIBUTION

The capability indexes in this case are calculated based on the assumption that the process data are normally distributed, and the process is stable and within control. Two sets of capability indexes are calculated: *Potential (within) Capability* and *Overall Capability*.

Potential Capability

- The potential or within capability indexes are: Cp, Cpl, Cpu, Cpk, and Ccpk

- These capability indexes are calculated based on the estimate of $\hat{\sigma}_{within}$ or the variation within each subgroup. If the data are in one column and the subgroup size is 1, this standard deviation is calculated based on the moving range (the adjacent observations are treated as subgroups). If the subgroup size is greater than 1, the within standard deviation is calculated using the range or standard deviation control chart (you can specify the method you want).
- …. the potential capability of the process tells what the process would be capable of producing if the process did not have shifts and drifts; or, how the process could perform relative to the specification limits …..(

Overall Capability

- The overall capability indexes are: Pp, Ppl, Ppu, Ppk, and Cpm
- These capability indexes are calculated based on the estimate of

  $\hat{\sigma}_{overall}$ or the overall variation, which is the variation of the entire data in the study.

- According to the MINITAB help screen, the overall capability of the process tells how the process is actually performing relative to the specification limits.

If there is a substantial difference between within and overall variation, it may be an indication that the process is out of control, or that the other sources of variation are not estimated by within capability [see MINITAB manual].

Note: According to some authors, Cp and Cpk assess the potential “short-term” capability using a “short-term” estimate of standard deviation, while Pp and Ppk assess overall or “long-term” capability using the “long-term” or overall standard deviation. Table 6.17 contains the formulas and their descriptions.

Formulas for the Process Capability using Normal Distribution

Table 6.17 shows the formulas for different process capability indexes.

Table 6.17

Capability Indexes for Potential (within) Process Capability

$C_p = \dfrac{USL - LSL}{6\hat{\sigma}_{within}}$	USL = upper specification limit LSL = lower specification limit $\hat{\sigma}_{within} =$ estimate of within subgroup standard deviation
$C_{PL} = \dfrac{\bar{x} - LSL}{3\hat{\sigma}_{within}}$	Ratio of the difference between process mean and lower specification limit to one-sided process spread $\bar{x} =$ process mean
$C_{PU} = \dfrac{USL - \bar{x}}{3\hat{\sigma}_{within}}$	Ratio of the difference between upper specification limit to one-sided process spread
$C_{PK} = Min.\{C_{PU}, C_{PL}\}$	Takes into account the shift in the process. The measure of C_{PK} relative to C_P is a measure of how off-center the process is. If $C_P = C_{PK}$ the process is centered; if $C_{PK} < C_P$ the process is off-center.
$CC_{PK} = \dfrac{USL - \hat{\mu}}{3\hat{\sigma}_{within}}$:	: :
$CC_{PK} = \dfrac{Min\{(USL - \hat{\mu}), (\hat{\mu} - LSL)\}}{3\hat{\sigma}_{within}}$	: :

Note: In all the above cases, the standard deviation is the estimate of *within subgroup* standard deviation. As noted above, the formulas for estimating standard deviation differs from case to case. It is very important to calculate the correct standard deviation. The standard deviation formulas are discussed later.

Relationship between Cp and Cpk

The index Cp determines only the spread of the process. It does not take into account the shift in the process. Cpk determines both the spread and the shift in the process.

:

and the relationship between Cp and Cpk is given by

$$C_{pk} = (1 - k)C_p$$

Note that Cpk never exceeds Cp. When Cpk=Cp, the process is centered midway between the specification limits. Both these indexes Cp and Cpk together provide information about how the process is performing with respect to the specification limits.

OVERALL PROCESS CAPABILITY INDICES (OR PERFORMANCE INDICES)

MINITAB also calculates the overall process capability indexes. These indexes are Pp, P_{PL}, P_{PU}, P_{pk}, and Cpm. The formulas for calculating these indexes are similar to those of potential capabilities except that the estimate of the standard deviation is an *overall standard deviation* and not within group standard deviation. The formulas for the overall capability indexes refer to Table 6.18.

Table 6.18
Capability Indices for Overall Process Capability

$C_p = \dfrac{USL - LSL}{6\hat{\sigma}_{within}}$	USL = upper specification limit LSL = lower specification limit $\hat{\sigma}_{overall}$= estimate of overall subgroup standard deviation This is the performance index that does not take into account the process centering. Continued…
$P_{PL} = \dfrac{\bar{x} - LSL}{3\hat{\sigma}_{overall}}$	Ratio of the difference between process mean and lower specification limit to one-sided process spread $\bar{x} =$ process mean
$P_{PU} = \dfrac{USL - \bar{x}}{3\hat{\sigma}_{overall}}$	: :

$P_{PK} = Min.\{P_{PL}, P_{PU}\}$	:
C_{pm}	This index is the ratio of (USL-LSL) to the square root of mean squared deviation from the target. This index is not calculated if the target value is not specified. A higher value of this index is an indication of a better process. This index is calculated for the known values of USL, LSL, and the target (T).
$C_{pm} = \dfrac{USL - LSL}{(tolerance) * \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - T)^2}{n-1}}}$	Used when USL, LSL, and target value, T are known T = target value = (USL+LSL)/2 = μ tolerance = 6 (sigma tolerance)
$C_{pm} = \dfrac{Min.\{(T - LSL), (USL - T)\}}{\dfrac{tolerance}{2} * \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - T)^2}{n-1}}}$: :	USL, LSL, and target, T are known but T $\neq$ (USL+LSL)/2

(Note: The above formulas are very similar to what MINITAB uses to calculate these indexes. See the MINITAB help screen for details).

In this section, we present several cases involving process capability analysis when the underlying process data are normally distributed. The process capability report and the analyses are presented for different cases.

Examples on Process Capability

Example 6.3

Calculate the capability indices - Cp, Cpl, Cpu, and Cpk for the process for which the data are given below. Interpret their meaning. Explain the difference between Cp and Cpk.

USL = 10.050, LSL = 9.950, $\hat{\mu} = 9.999$ and $\hat{\sigma} = 0.0165$ as estimates of μ and σ ($\hat{\mu}$ is the same as $\bar{x}$).

Solution:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{10.050 - 9.950}{6(0.0165)} = 1.01$$

(Note; $\hat{\sigma}$ is the estimate of standard deviation).

$$C_{PL} = \frac{\bar{x} - LSL}{3\hat{\sigma}} = \frac{9.999 - 9.950}{3(0.0165)} = 0.99$$

$$C_{PU} = \frac{USL - \bar{x}}{3\hat{\sigma}} = \frac{10.050 - 9.999}{3(0.0165)} = 1.03$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{0.99, 1.03\} = 0.99$$

Cp=1.01 means that the process is marginally capable (just able to meet the specifications). Cp=Cpk means that the process is centered. This process is slightly off-centered.

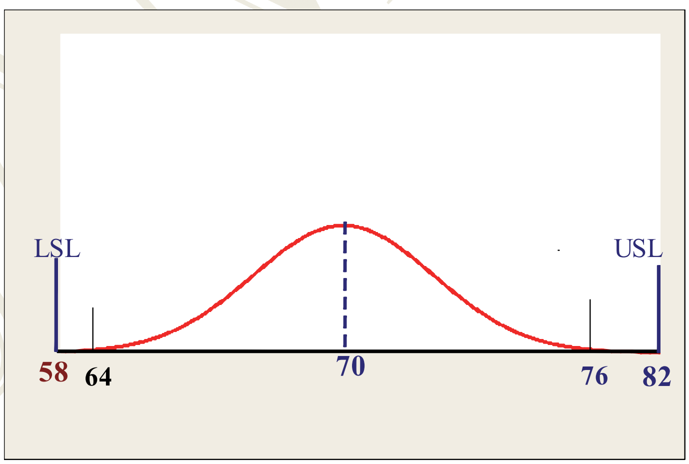
Difference between Cp and Cpk: The process capability ratio or Cp does not take into account the shift in the process mean. It does not consider where the mean is relative to the specifications. Cp measures only the spread of the specifications relative to the 6-sigma spread or the process spread. Cpk on the other hand, ….

Example 6.4

(a) Given $\bar{x} = 70$ and $\hat{\sigma} = 2$ (as estimates of μ and σ), LSL =58, USL = 82. Calculate the process capability indices: Cp, Cpl, Cpu, and Cpk.

Solution: The problem is visually shown in Figure 6.20.

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{82 - 58}{6(2)} = 2.0$$



$$C_{PL} = \frac{\bar{x} - LSL}{3\hat{\sigma}} = \frac{70 - 58}{3(2)} = 2.0 \qquad C_{PU} = \frac{USL - \bar{x}}{3\hat{\sigma}} = \frac{82 - 70}{3(2)} = 2.0$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{2.0, 2.0\} = 2.0$$

(b) Calculate the capability indices for the data in part (a) if the mean has shifted from 70 to 73 (all the other values are same as in part (a)).

:

:

Example 6.5

Complete Table 6.18 by calculating the Cp, Cpk, and the process fallout, or the defects in parts per million (PPM) for different sigma levels. Calculate ratios Cp, Cpk, and the process fall out when the process is centered and when there is a shift of 1.5-sigma. Note: Cpk is calculated based on an assumed shift of 1.5σ. Some calculations for 3σ process are shown in the table. Complete the rest of the table.

Table 6.18: Calculation of Defects (PPM) for Different Sigma Level

Sigma Level	Process Centered?	C_p	C_{pk}	Process Fallout (Defects in PPM)
$\pm 3\sigma$	Yes	1.0	1.0	2700 PPM
	No	1.0	0.5	66,811 PPM
$\pm 4\sigma$	Yes			
	No			
$\pm 4.5\sigma$	Yes			
	No			
$\pm 6\sigma$	Yes			
	No			

***Solution:* _Calculations for ±3σ_**

Curve A in Figure 6.22 shows a 3σ process. Curve B shows a shift of 1.5σ in the mean. The Cp, Cpk, and the process fallout (in defective PPM) can be calculated as shown below.

<u>3σ Centered Process</u>

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{6\sigma}{6\sigma} = 1.0$$

To calculate the process fallout or the defective, calculate P(Z>3.0) using the standard normal curve. This can be calculated using MINITAB (see instructions in Table 6.19).

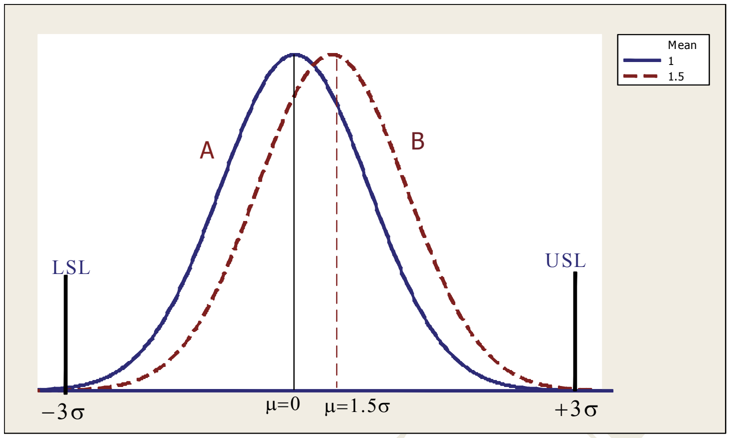


Figure 6.22: A 3-Sigma Process with a Shift of 1.5-Sigma

Table 6.19

In MINITAB, execute the following commands

Calc ➢Probability Distributions ➢ Normal

Complete the Normal Distribution dialog box click the circle to the left of **Cumulative probability** and type the following in the boxes:

Mean	**0.0**
Standard deviation	**1.0**
Input constant	**3.0**

Click **OK.**

The session screen will show the result below.

Cumulative Distribution Function

```
Normal with mean = 0 and standard deviation = 1

x     P( X <= x )
3      0.998650
```

Using the above result, $P(z > +3) = 1 - 0.998650 = 0.00135$

and, $P(z > -3) = 0.00135$

Therefore, defective in a million (PPM) for a ±3σ process=(2*0.00135)*10^6 = 2700. This process fallout is for a centered process. It indicates that this process would produce 2700 defective products out of a million.

Cp, Cpk, and the process fallout (in defective PPM) for a shift of +1.5σ

Curve B shows a shift of 1.5σ in the mean. The Cp, Cpk, and the process fallout (in defective PPM) can be calculated as:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{6\sigma}{6\sigma} = 1.0$$

$$C_{PU} = \frac{USL - \mu}{3\sigma} = ... \quad ; \quad C_{PL} = \frac{\mu - LSL}{3\sigma} = ... = 0.5$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{0.5, 0.5\} = 0.5$$

To calculate the process fallout or the defective, calculate P(Z>3.0) using the standard normal curve with a mean of 1.5. This can be calculated using MINITAB. Follow the instructions in Table 6.19 but type 1.5 for the mean. The result is shown below.

Cumulative Distribution Function

```
Normal with mean = 1.5 and standard deviation = 1
x  P( X <= x )
3     0.933193
```

Using the above result, $P(z > +3) = 1 - 0.933193 = 0.0668072$

The total process fallout or defective (in PPM) for a shift of $\pm 1.5\sigma$ would be $0.066811*10^6$ (6.68072+0.0003)% or, 66,811 in a million. This value is usually reported as 66,807 in the literature. Table 6.20 shows the Cp, Cpk, and the defective (in PPM) for 4σ, 4.5σ and 6σ processes.

<u>Calculations for a ±4σ Process with a Shift of ±1.5σ</u>

Figure 6.23 shows a 4-Sigma process with a shift of 1.5-sigma on either side. The Cp, Cpk, and the process fallout (in defective PPM) for this process are calculated below.

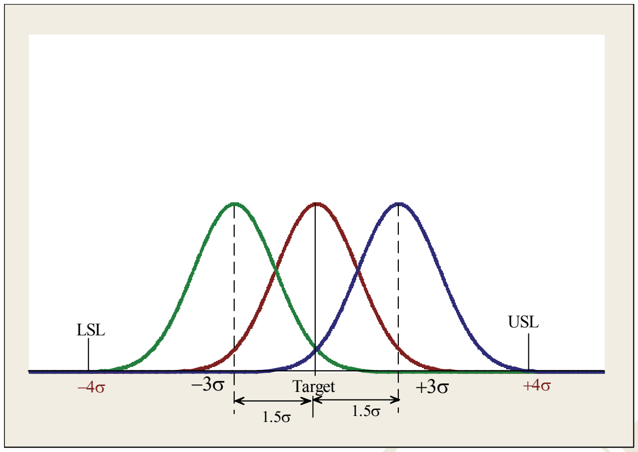


Figure 6.23: A 4-Sigma Process with a Shift of 1.5-Sigma

When the process is centered:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{8\sigma}{6\sigma} = 1.33$$

To calculate the process fallout or the defective, calculate P(Z>4.0) using the standard normal curve. The instructions are the same as in Table 6.19 except the following —....

```
Cumulative Distribution Function
Normal with mean = 0 and standard deviation = 1

x  P( X <= x )
4     0.999968
```

From the above table, $P(z > +4) = 1 - 0.999968 = 0.000032$

:

:

:

Cp, Cpk, and the process fallout (in defective PPM) for a shift of +1.5σ

Figure 6.23 shows a shift of 1.5 σ in the mean for a 4-sigma process. The Cp, Cpk, and the process fallout (in defective PPM) can be calculated as:

$$C_{PU} = \frac{USL - \mu}{3\sigma} = \frac{4\sigma - 1.5\sigma}{3\sigma} = 0.83$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{..., 0.83\} = 0.83$$

To calculate the process fallout or the defective, calculate P(Z>4.0) using the standard normal curve with a mean of 1.5. This can be calculated using MINITAB. Follow the instructions in Table 6.19 but type a 1.5 for the mean and 4.0 in the mean box. The result is shown below.

Cumulative Distribution Function
```
Normal with mean = 1.5 and standard deviation = 1
x  P( X <= x )
4     0.993790
```

From the above table,

$$P(z > +4) = 1 - 0.993790 = 0.00621$$

Therefore, the process fallout or defective (in PPM) for a shift of +1.5 σ would be $0.00621*10^6$ or, 6210. Similarly, the defective (in PPM) for a shift of -1.5 σ would also be 6210.

Table 6.20 shows the Cp, Cpk, and the defective (in PPM) for 3 σ, 4 σ, 4.5 σ, and 6 σ processes. Note that the defects in PPM corresponding to the Cpk values are calculated for only 1.5σ shift in the positive direction. For ±1.5σ shift, the process fall out values will be twice the values shown in Table 6.20. You should verify these results.

Table 6.20: Cp, Cpk, and Process Fallout in PPM for Different Sigma Levels

Sigma Level	***Process Centered?***	C_p	C_{pk}	***Process Fallout (Defects in PPM)***
$\pm 3\sigma$	Yes	1.0	1.0	2700
	No	1.0	0.5	66,807
$\pm 4\sigma$	Yes	1.33	:	:
	No			:
$\pm 4.5\sigma$	Yes			:
	No	1.5	:	:
$\pm 6\sigma$	Yes		:	:
	No	2.0	1.5	3.4

Example 6.1- Calculating Process Capability using Control Charts

A chemical company manufactures and markets 50 lb. nitrogen fertilizer for the lawns. Due to some recent problems in their production process, overfilling and under filling in the filling bags of fertilizer have been reported. The problem was investigated and appropriate adjustments were made to the machines that are used to fill the fertilizer bags. When the process was believed to be stable, the quality supervisor collected 30 samples each of size 5. The control charts for $\bar{x}$ and R were constructed and the tests were conducted for the special or assignable causes. The process was found to be within control and no assignable causes were present. The $\bar{x}$ and R control charts for the process are shown in Figure 6.15. Determine the process capability for this process based on the average of range value, or $\bar{R}$ reported on the R chart. Note that $\bar{R}$ - the average of all subgroup range can be used to estimate the process standard deviation.

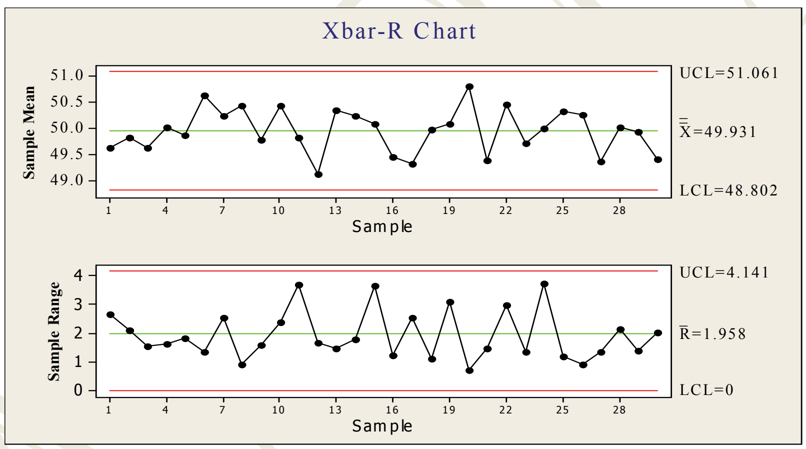


Figure 6.15: The $\bar{x}$ and R Control Charts for Example 6.1

Solution:

First, estimate the standard deviation, σ from the given information using:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \ldots = 0.842$$

Note: $\hat{\sigma}$ is the estimate of σ . *The value of* $\bar{R}$ *is reported in the chart for range in Figure 6.15 and* d_2 *is obtained from the table - 'factors for computing 3σ control limits' in Appendix B (Table B.3). The value of* d_2 .....

The process capability,

$$6\hat{\sigma} = 6(0.842) = 5.052$$

(a) The process capability can also be determined by estimating the $\hat{\sigma}$ using the average of standard deviations, $\bar{s}$ of the subgroups instead of average of the subgroup range. Here we demonstrate the calculation of process capability using the standard deviation.

Figure 6.16 shows the $\bar{x} - S$ - control charts for the average and standard deviation.

Determine the process capability using the value of $\bar{s}$ in the control chart.

First, estimate the standard deviation, σ from the given information using:

$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \ldots = 0.841$$

Note: $\hat{\sigma}$ is the estimate of σ . *The value of $\bar{s}$ is reported in the chart for standard deviation (s)…. and c_4 is obtained from the table - 'factors for computing…*

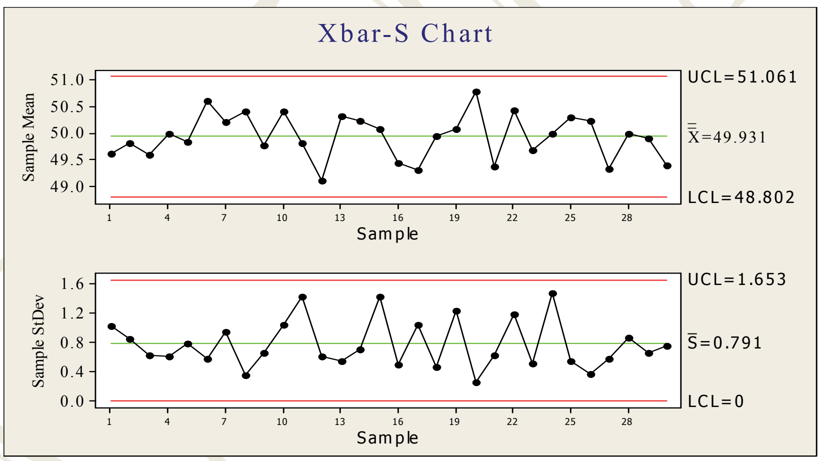


Figure 6.16: The $\bar{x}$ and S- control charts

The process capability: $6\hat{\sigma} = 5.046$

Continued…

CASE 1 : PROCESS CAPABILITY ANALYSIS (USING NORMAL DISTRIBUTION)

In this case, we want to assess the process capability for a production process that produces certain type of pipe. The inside diameter of the pipe is of concern. The specification limits on the pipes are 7.000 ± 0.025 cm. There has been a consistent problem with meeting the specification limits, and the process produces a high percentage of rejects. The data on the diameter of the pipes were collected. A random sample of 150 pipes was selected. The measured diameters are shown in the data file **PCA2.MTW** (column C1).

The process producing the pipes is stable. The histogram and the probability plot of the data show that the measurements follow a normal distribution. The variation from pipe-to-pipe can be estimated using the within group standard deviation or $\hat{\sigma}_{within}$. Since the process is stable and the measurements are normally distributed, the normal distribution option of process capability analysis can be used.

PROCESS CAPABILITY OF PIPE DIAMETER (PRODUCTION RUN 1)

To assess the process capability for the first sample of 150 randomly selected pipes, follow the steps in Table 6.19. Note that the data are in one column (column C1 of the data file) and the subgroup size is one.

Table 6.19

PROCESS CAPABILITY ANALYSIS	Open the data file **PCA2.MTW** Use the command sequence **Stat ➢Quality Tools ➢ Capability Analysis ➢Normal** In the **Data are Arranged as** section, click the circle next to **Single column** and select or type **C1 PipeDia:Run 1** in the box Type **1** in the **Subgroup size** box In the **Lower spec.** and **Upper spec.** boxes, type **6.975** and **7.025** respectively : : Click **OK** Click the **Options** tab on the upper right corner Type **7.000** in the **Target (adds Cpm to table)** box In the **Calculate statistics using** box a **6** should show by default Under **Perform Analysis**, **Within subgroup analysis** and **Overall analysis** boxes should be checked (you may uncheck the analysis

	not desired) Under **Display**, select the options you desire (some are checked by default) Type a title if you want or a default title will be provided Click **OK** in all dialog boxes.

The process capability report as shown in Figure 6.11 will be displayed.

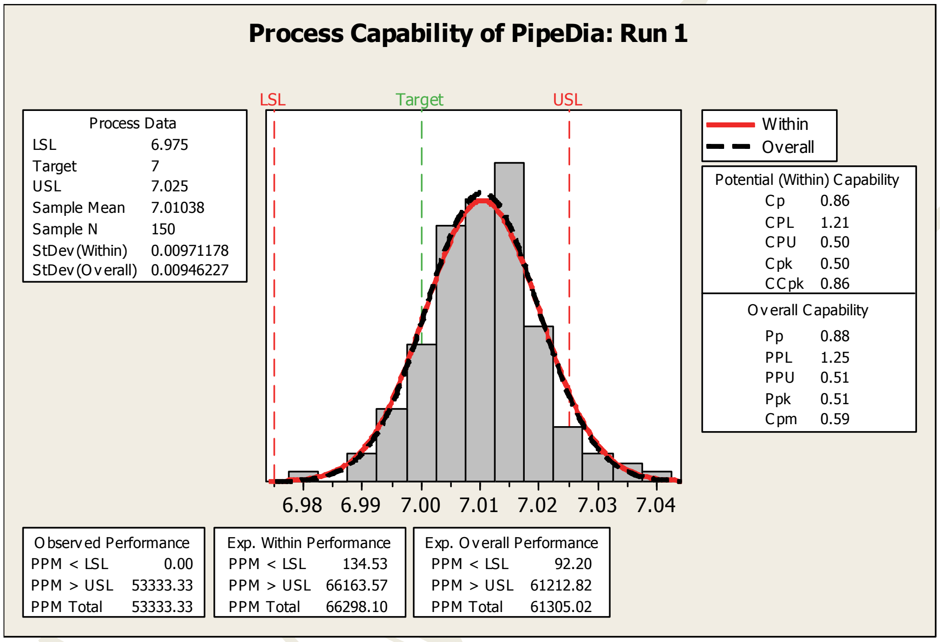


Figure 6.11: Process Capability Report of Pipe Diameter: Run 1

INTERPRETING THE RESULTS

1. The upper left box reports the process data including the lower specification limit, target, and the upper specification limit. These values were provided by the program. The calculated values are the process sample mean and the estimates of within and overall standard deviations.

Process Data	
LSL	6.975
Target	7
USL	7.025
Sample Mean	7.01038
Sample N	150
StDev(Within)	0.00971178
StDev(Overall)	0.00946227

2. The report in Figure 6.11 shows the histogram of the data along with two normal curves overlaid on the histogram. One normal curve (with a solid line) ……..

3. The histogram and the normal curves can be used to check visually if the process data are normally distributed. To interpret the process capability, the normality assumption must hold. In Figure 6.11, ……

4. There is a deviation of the process mean (7.010) from the target value of 7.000. Since the process mean is greater than the target value, the pipes produced by this process exceed the upper specification limit (USL). A significant percentage of the pipes are outside of ………………..

5. The **potential or within process capability** and the **overall capability** of the process is reported on the right hand side. For our example, the values are

Potential (Within) Capability	
Cp	0.86
CPL	1.21
CPU	0.50
Cpk	0.50
CCpk	0.86
Overall Capability	
Pp	0.88
PPL	1.25
PPU	0.51
Ppk	0.51
Cpm	0.59

:

6. The value of Cp=0.86 indicates that the process is not capable (Cp < 1). Also, Cpk = 0.50 is less than Cp=0.86. This means that the process is off-centered. Note that when Cpk = Cp then the process ……

7. Cpk=0.50 (less than 1) is an indication that an improvement in the process is warranted.

:

:

8. Higher value of Cpk indicates that the process is meeting the target with minimum process variation. If the process is off-centered, Cpk value is smaller compared to Cp even …………………

9. The **overall capability indexes** or the process performance indexes Pp, PPL, PPU, Ppk, and Cpm are also calculated and reported. Note that these indexes are based on the estimate of **overall standard deviation……….**

10. Pp and Ppk have similar interpretation as Cp and Cpk. For this example, note that Cp and Cpk values (0.86 and 0.50 respectively) are very close to Pp and Ppk (0.88 and 0.51). When Cpk equals Ppk then the within subgroup standard deviation is ….

11. The index Cpm is calculated for the specified target value. If no target value is specified, Cpm …..

12. …………….For this process, Pp = 0.88, Ppk = 0.51, and Cpm = 0.59. A comparison of these values indicates that the process is off-center.

13. The bottom three boxes report observed performance, expected within performance, and expected overall process performance in parts per million (PPM). The **observed performance** box in Figure 6.11 shows the following values:

Observed Performance	
PPM < LSL	0.00
PPM > USL	53333.33
PPM Total	53333.33

This means that the number of pipes below the lower specification limit (LSL) is zero; that is, …..

The **expected "within" performance** is based on the estimate of within subgroup standard deviation. These are the average number of parts below and above the specification limits in parts per million. The values are calculated using the following formulas

$$P\left[z < \frac{(LSL - \bar{x}}{\hat{\sigma}_{within}}\right] * 10^6$$ for the expected number below the LSL

$$P\left[z > \frac{(USL - \bar{x}}{\hat{\sigma}_{within}}\right] * 10^6$$ for the expected number above the USL

note that $\bar{x}$ is the process mean.

For this process, the **Expected Within Performance** measures are

Exp. Within Performance	
PPM < LSL	134.53
PPM > USL	66163.57
PPM Total	66298.10

The above values show the ….

14. The Expected Overall Performance is calculated using similar formulas as in within performance, except the estimate of standard deviation is based on overall data.

For this process, the **Expected Overall Performance** measures are

Exp. Overall Performance	
PPM < LSL	92.20
PPM > USL	61212.82
PPM Total	61305.02

These values are based on …..continued…

CASE 3: PROCESS CAPABILITY OF PIPE DIAMETER (PRODUCTION RUN 3)

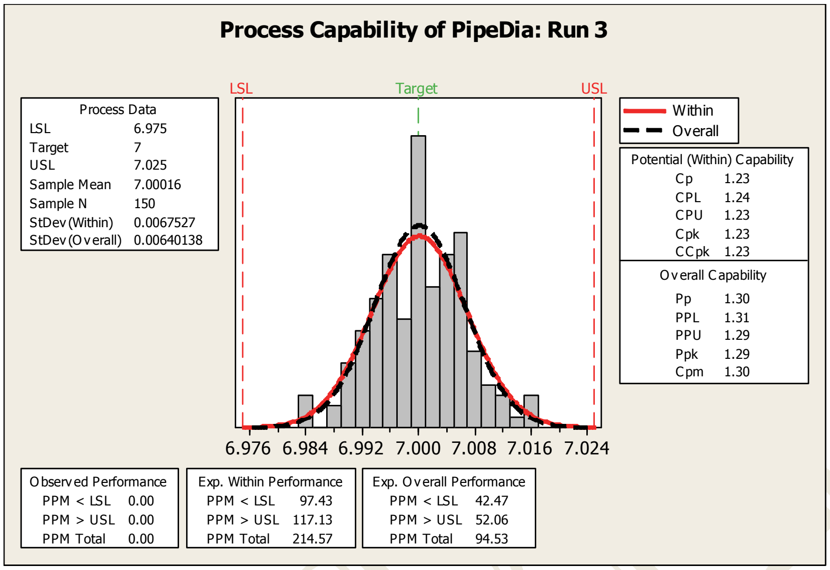


Figure 6.13: Process Capability Report of Pipe Diameter: Run3

CASE 4: PROCESS CAPABILITY ANALYSIS OF PIZZA DELIVERY

A Pizza chain franchise advertises that any order placed through a phone or the internet will be delivered in 15 minutes or less. If the delivery takes more than 15 minutes, there is no charge and the delivery is free. This offer is available within a radius of 3 miles from the delivery location.

In order to meet the delivery promise, the Pizza chain has set a target of 12 ± 2.5 minutes….

Using the 100 delivery times (shown in Column 1 of data file **PAC3.MTW**), a process capability analysis was conducted. To run the process capability, follow the instructions in Table 6.20. The process capability report is shown in Figure 6.14.

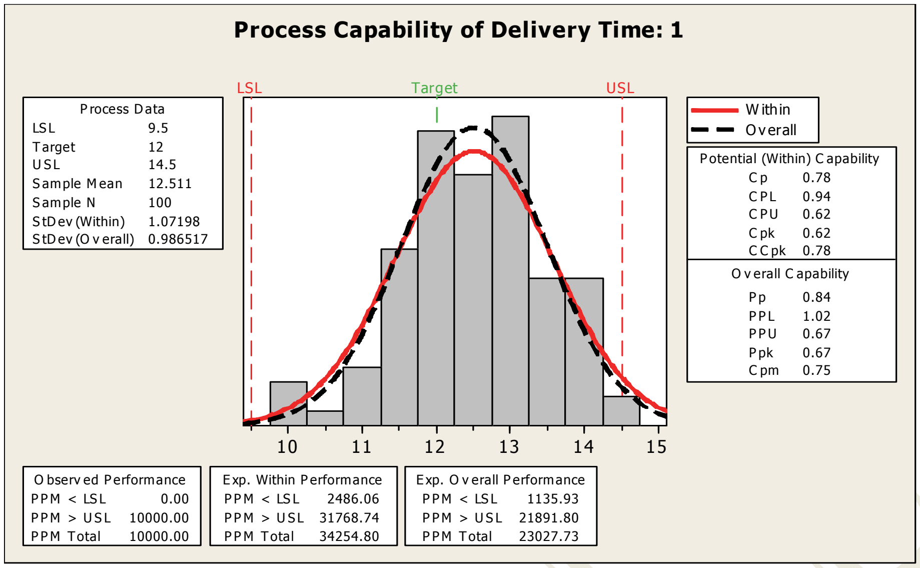


Figure 6.14: Process Capability Report of Pizza Delivery Time: 1 :

:

CASE 6: PERFORMING PROCESS CAPABILITY ANALYSIS WHEN THE PROCESS MEASUREMENTS DO NOT FOLLOW A NORMAL DISTRIBUTION (NON-NORMAL DATA)

The process capability report is shown in Figure 6.21.

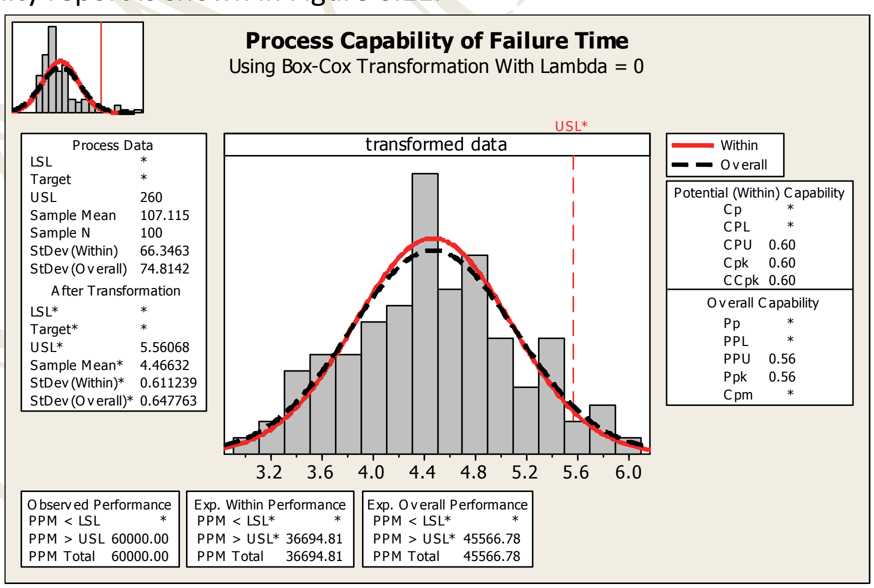
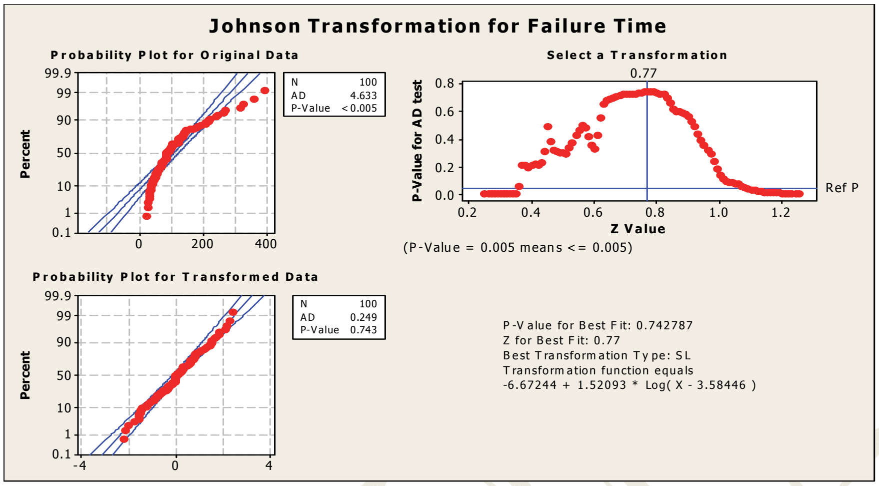


Figure 6.21: Process Capability Report of Failure Time Data using ***Box-Cox Transformation***

PROCESS CAPABILITY OF NON-NORMAL DATA USING JOHNSON TRANSFORMATION



PROCESS CAPABILITY USING DISTRIBUTION FIT

The other way of determining the process capability of non-normal data is to use a distribution fit approach. In cases where data are not normal, fit an appropriate distribution and use that distribution-rather than a normal distribution-to determine the process capability. We will illustrate the method using an example.

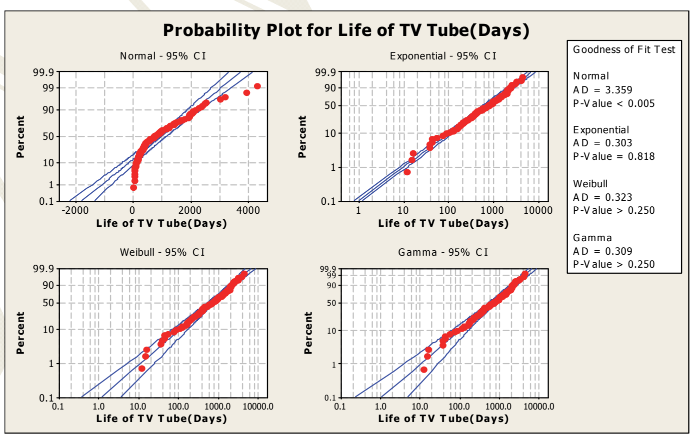


Figure 6.26: Probability Plots for Selected Distribution

PROCESS CAPABILITY SIX-PACK

Another option available for process capability analysis is *process capability six-pack*. The process capability using this option displays

- A $\bar{x}$ chart (or individual chart for subgroup means),
- An R-chart (S chart for a subgroup size greater than 8),
- A run chart or moving range chart,
- A histogram,
- A normal probability plot of the process data to check the normality, and
- The between/within statistics and overall capability indexes.

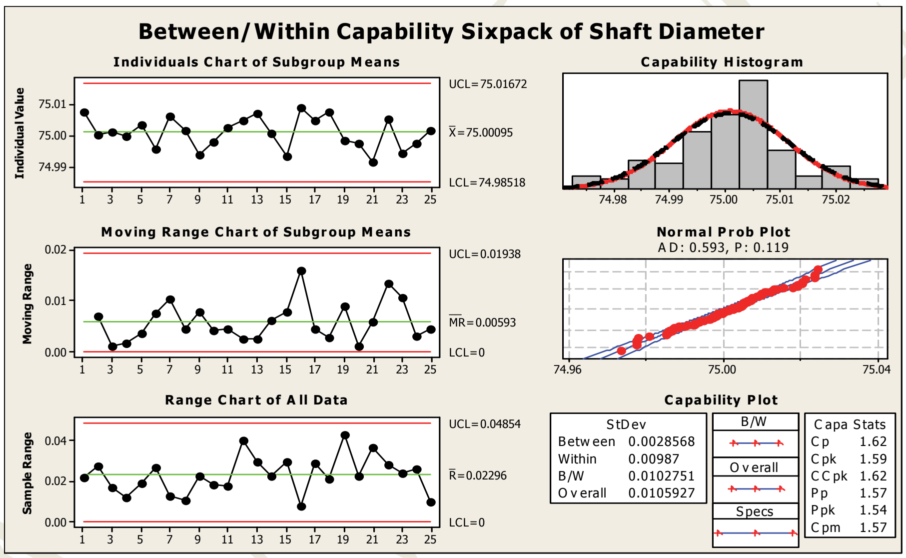


Figure 6.34: Process Capability Six-pack of Shaft Diameter

INTERPRETING THE RESULTS

The report shows the control charts for Xbar and R. The tests for special causes are conducted and reported on the session screen. No special causes were found indicating that the process is stable and in control. The capability histogram shows that the……………………

Chapter 6 of Six Sigma Volume 1 contains detailed analysis and interpretation of process capability analysis with data files and step-wise computer instructions for both normal and non-normal data.

To buy chapter 6 or Volume I of Six Sigma Quality Book, please click on our products on the home page.