Chapter 6

Process Capability Analysis for Six Sigma

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Master Black Belt
Chapter 6: Process Capability Analysis for Six Sigma

CHAPTER HIGHLIGHTS

This chapter deals with the concepts and applications of process capability analysis in Six Sigma. Process Capability Analysis is an important part of an overall quality improvement program. Here we discuss the following topics relating to process capability and Six Sigma:

1. Process capability concepts and fundamentals
2. Connection between the process capability and Six Sigma
3. Specification limits and process capability indices
4. Short-term and long-term variability in the process and how they relate to process capability
5. Calculating the short-term or long-term process capability
6. Using the process capability analysis to:
   - assess the process variability
   - establish specification limits (or, setting up realistic tolerances)
   - determine how well the process will hold the tolerances (the difference between specifications)
   - determine the process variability relative to the specifications
   - reduce or eliminate the variability to a great extent
7. Use the process capability to answer the following questions:
   - Is the process meeting customer specifications?
   - How will the process perform in the future?
   - Are improvements needed in the process?
   - Have we sustained these improvements, or has the process regressed to its previous unimproved state?
8. Calculating process capability reports for normal and non-normal data using MINITAB.

CHAPTER OUTLINE

Process Capability
Process Capability Analysis
Determining Process Capability
   Important Terms and Their Definitions
   Short-term and Long-term Variations
Process Capability Using Histograms
Process Capability Using Probability Plot
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Estimating Percentage Nonconforming for Non-normal Data: Example 1
Estimating Nonconformance Rate for Non-normal Data: Example 2
Capability Indexes for Normally Distributed Process Data
Determining Process Capability Using Normal Distribution
Formulas for the Process Capability Using Normal Distribution
Relationship between Cp and Cpk
The Percent of the Specification Band used by the Process
Overall Process Capability Indexes (or Performance Indexes)
Calculating Process Capability
  Case 1: Process Capability Analysis (Using Normal Distribution)
  Case 2: Process Capability of Pipe Diameter (Production Run 2)
  Case 3: Process Capability of Pipe Diameter (Production Run 3)
  Case 4: Process Capability Analysis of Pizza Delivery
  Case 5: Process Capability Analysis: Data in One Column (Subgroup size=1)
    (a) Data Generated in a Sequence, (b) Data Generated Randomly
  Case 6: Performing Process Capability Analysis: When the Process
    Measurements do not follow a Normal Distribution
    Process Capability using Box Cox Transformation
Process Capability of Non-normal Data Using Box-Cox Transformation
Process Capability of Nonnormal Data Using Johnson’s Transformation
Process Capability Using Distribution Fit
Process Capability Using Control Charts
Process Capability Using x-bar and R Chart
Process Capability SixPack
Process Capability Analysis of Multiple Variables Using Normal Distribution
Process Capability Analysis Using Attribute Charts
  Process Capability Using a p-Chart
  Process Capability Using a u-Chart
Notes on Implementation
Hands-on Exercises

This document contains explanation and examples on process capability analysis from Chapter 6 of our Six Sigma Volume 1. The book contains numerous cases, examples and step-wise computer instruction with data files.
Process Capability

Process Capability is the ability of the process to meet specifications. The capability analysis determines how the product specifications compare with the inherent variability in a process. The inherent variability of the process is the part of process variation due to common causes. The other type of process variability is due to the special causes of variation.

It is a common practice to take the six-sigma spread of a process’s inherent variation as a measure of process capability when the process is stable. Thus, the process spread is the process capability, which is equal to six sigma.

Process Capability Analysis: An Important Part of an Overall Quality Improvement Program

The purpose of the process capability analysis involves assessing and quantifying variability before and after the product is released for production, analyzing the variability relative to product specifications, and improving the product design and manufacturing process to reduce the variability. Variation reduction is the key to product improvement and product consistency.

The process capability analysis is useful in determining how well the process will hold the tolerances (the difference between specifications). The analysis can also be useful in selecting or modifying the process during product design and development, selecting the process requirements for machines and equipment, and above all, reducing the variability in production processes.

Determining Process Capability

The following points should be noted before conducting a process capability analysis.

- Process capability should be assessed once the process has attained statistical control. This means that the special causes of variation have been identified and eliminated. Once the process is stable, ............
- In calculating process capability, the specification limits are required in most cases, ...... Unrealistic or inaccurate specification limits may not provide correct process capability.
Process capability analysis using a histogram or a control chart is based on the assumption that the process characteristics follow a normal distribution. While the assumption of normality holds in many situations, there are cases where the processes do not follow a normal distribution. Extreme care should be exercised where normality does not hold.

Short-term and Long-term Variation

The standard deviation that describes the process variation is an integral part of process capability analysis. In general, the standard deviation is not known and must be estimated from the process data. The estimated standard deviation used in process capability calculations may address "short-term" or "long-term" variability. The variability due to common causes is described as "short-term" variability, while the variability due to special causes is considered "long-term" variability.

Some examples of "long-term" variability may be lot-to-lot variation, operator-to-operator variation, day-to-day variation or shift-to-shift variation. Short-term variability may be within-part variation, part-to-part variation, variations within a machine, etc. However, the literature differs on what is "long-term" and what is "short-term" variation.

In process capability analysis, both "short-term" and "long-term" indexes are calculated and are not considered separately in assessing process capability. The indexes Cp and Cpk are "short-term" capability indexes and are calculated using "short-term" standard deviation whereas, Pp and Ppk are "long-term" capability and are calculated using "long-term" standard deviation estimate. These are discussed in more detail later.

Determining Process Capability

Following are some of the methods used to determine the process capability. The first two are very common and are described below.

1. Histograms and probability plots,
2. Control charts, and
3. Design of experiments.

Process Capability Using Histograms: Specification Limits Known
Suppose that the specification limits on the length is 6.00±0.05. We now want to determine the percentage of the parts outside of the specification limits. Since the measurements are very close to normal, we can use the normal distribution to calculate the nonconforming percentage. Figure 6.2 shows the histogram of the length data with the target value and specifications limits. To do this plot, follow the instructions in Table 6.2.

### Table 6.2

<table>
<thead>
<tr>
<th>HISTOGRAM WITH SPECIFICATION LIMITS</th>
<th>Open the worksheet PCA1.MTW</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the main menu, select Graph ➤ Histogram</td>
<td></td>
</tr>
<tr>
<td>Click on With Fit then click OK</td>
<td></td>
</tr>
<tr>
<td>For Graph variables, ............Click the Scale then click the Reference Lines tab</td>
<td></td>
</tr>
<tr>
<td>In the Show reference lines at data values type 5.95 6.0 6.05</td>
<td></td>
</tr>
<tr>
<td>Click OK in all dialog boxes.</td>
<td></td>
</tr>
</tbody>
</table>

![Histogram of Length](image1)

**Figure 6.2:** Histogram of the Length Data with Specification Limits and Target

![Fitted Normal Curve with Reference Line](image2)

**Figure 6.3:** Fitted Normal Curve with Reference Line for the Length Data
Table 6.4

<table>
<thead>
<tr>
<th>Cumulative Distribution Function</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal with mean = 5.999 and standard deviation = 0.0199</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>P( X &lt;= x )</td>
</tr>
<tr>
<td>5.95</td>
<td>0.0069022</td>
</tr>
<tr>
<td>6.05</td>
<td>0.994809</td>
</tr>
</tbody>
</table>

From the above table, the percent conforming can be calculated as 0.994809 - 0.0069022 = 0.98790 or, 98.79%. Therefore, the percent outside of the specification limits is 1 - 0.98790 or, 0.0121 (1.21%).

**PROCESS CAPABILITY USING PROBABILITY PLOTS**

A probability plot can be used in place of a histogram to determine the process capability. Recall that a probability plot can be used to determine the distribution and shape of the data. If the probability plot indicates that the distribution is normal, the mean and standard deviation can be estimated from the plot. For the length data discussed above, we know that the distribution is normal. We will construct a probability plot (or perform a Normality test) .......

Table 6.5

<table>
<thead>
<tr>
<th>NORMALITY TEST USING PROBABILITY PLOT</th>
<th>Open the worksheet PCA1.MTW</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the main menu, select Stat ➤ Basic Statistics ➤ Normality Test</td>
<td></td>
</tr>
<tr>
<td>For Variable, Select C1 Length</td>
<td></td>
</tr>
<tr>
<td>Under Percentile Line, .....type 50 84 in the box</td>
<td></td>
</tr>
<tr>
<td>Under Test for Normality, .....Anderson Darling</td>
<td></td>
</tr>
<tr>
<td>Click OK</td>
<td></td>
</tr>
</tbody>
</table>

: |

: |

For a normal distribution, the mean equals median, which is 50th percentile, and the standard deviation is the difference between the 84th and 50th percentile. From Figure 6.4, the estimated mean is 5.9985 or 5.999 and the estimated standard deviation is

\[ \sigma = 84^{th} \text{ percentile} - 50^{th} \text{ percentile} = 6.0183 - 5.9985 = 0.0198 \]

Note that the estimated standard deviation is very close to what we got from earlier analysis. The process capability can now be determined as explained in the previous example.
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ESTIMATING PERCENTAGE NONCONFORMING FOR NON-NORMAL DATA: EXAMPLE 1

When the data are not normal, an appropriate distribution should be fitted to the data before calculating the nonconformance rate.

Data file \texttt{PCA1.MTW} shows the life of a certain type of light bulb. The histogram of the data is shown in Figure 6.5. To construct this histogram, follow the steps in Table 6.1.

![Figure 6.5: Life of the Light bulb](image-url)
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For the data in Figure 6.5, it is required to calculate the capability with a lower limit because the company making the bulbs wants to know the minimum survival rate. They want to determine the percentage of the bulbs surviving 150 hours or less.

The plot in Figure 6.5 clearly indicates that the data are not normal. Therefore, if we use the normal distribution to calculate the nonconformance rate, it will lead to a wrong conclusion. It seems reasonable to assume that the life data might follow an exponential distribution. We will fit an exponential distribution to the data...

<table>
<thead>
<tr>
<th>Table 6.6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FITTING DISTRIBUTION (EXPONENTIAL)</strong></td>
</tr>
<tr>
<td>From the main menu, select <strong>Graph &gt; Histogram</strong></td>
</tr>
<tr>
<td>Click on <strong>With Fit</strong> then click <strong>OK</strong></td>
</tr>
<tr>
<td>Click the <strong>Distribution</strong> tab, check the box next to <strong>Fit Distribution</strong></td>
</tr>
<tr>
<td>Click the downward arrow and select <strong>Exponential</strong></td>
</tr>
<tr>
<td>Click <strong>OK</strong> in all dialog boxes.</td>
</tr>
</tbody>
</table>

The histogram with a fitted exponential curve shown in Figure 6.6 will be displayed. The exponential distribution seems to provide a good fit to the data. The parameter of the exponential distribution (mean = 494.1 hrs) is....

![Exponential Distribution Fitted to the Life Data](image)

Figure 6.6: Exponential Distribution Fitted to the Life Data

The plot using the exponential distribution clearly fits the data as most of the plotted plots are along the straight line. To do the probability plots in Figure 6.7, follow the instructions in Table 6.7.

| Table 6.7 |
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PROBABILITY PLOT

Open the worksheet PCA1.MTW
From the main menu, select Graph ➔ Probability Plots
Click on Single then click OK
For Graph variables, type C2 or select Life in Hrs
Click on Distribution then select Normal ...Click OK in all dialog boxes.

Figure 6.7: Probability Plots of the Life Data

Table 6.9

Cumulative Distribution Function
Exponential with mean = 494.1
\[ x \quad P(X \leq x) \]
\[ 150 \quad 0.261831 \]

The table shows the calculated probability, \( P(x \leq 150) = 0.2618 \). This means that 26.18% of the products will fail within 150 hours or less.

CAPABILITY INDICES FOR NORMALLY DISTRIBUTED PROCESS DATA
MINITAB provides several options for determining the process capability. The options can be selected by using the command sequence Stat > Quality Tools > Capability Analysis. This provides several options for performing process capability analysis including the following:

- Normal
- Between/Within
- Non-normal
- Multiple Variables (Normal)
- Multiple Variables (Nonnormal)
- Binomial
- Poisson

**Determining Process Capabilities Using Normal Distribution**

The capability indexes in this case are calculated based on the assumption that the process data are normally distributed, and the process is stable and within control. Two sets of capability indexes are calculated: Potential (within) Capability and Overall Capability.

**Potential Capability**

- The potential or within capability indexes are: Cp, Cpl, Cpu, Cpk, and Cc(pk)
These capability indexes are calculated based on the estimate of $\sigma_{within}$ or the variation within each subgroup. If the data are in one column and the subgroup size is 1, this standard deviation is calculated based on the moving range (the adjacent observations are treated as subgroups). If the subgroup size is greater than 1, the within standard deviation is calculated using the range or standard deviation control chart (you can specify the method you want).

... the potential capability of the process tells what the process would be capable of producing if the process did not have shifts and drifts; or, how the process could perform relative to the specification limits.

Overall Capability

- The overall capability indexes are: Pp, Ppl, Ppu, Ppk, and Cpm
- These capability indexes are calculated based on the estimate of $\sigma_{overall}$ or the overall variation, which is the variation of the entire data in the study.

According to the MINITAB help screen, the overall capability of the process tells how the process is actually performing relative to the specification limits.

If there is a substantial difference between within and overall variation, it may be an indication that the process is out of control, or that the other sources of variation are not estimated by within capability [see MINITAB manual].

Note: According to some authors, Cp and Cpk assess the potential “short-term” capability using a “short-term” estimate of standard deviation, while Pp and Ppk assess overall or “long-term” capability using the “long-term” or overall standard deviation. Table 6.17 contains the formulas and their descriptions.

FORMULAS FOR THE PROCESS CAPABILITY USING NORMAL DISTRIBUTION

Table 6.17 shows the formulas for different process capability indexes.
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Table 6.17
Capability Indexes for Potential (within) Process Capability

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
</table>
| $C_p = \frac{USL - LSL}{6\sigma_{within}}$ | $USL = \text{upper specification limit}$  
$LSL = \text{lower specification limit}$  
$\sigma_{within} = \text{estimate of within subgroup standard deviation}$ |
| $C_{PL} = \frac{\bar{x} - LSL}{3\sigma_{within}}$ | Ratio of the difference between process mean and lower specification limit to one-sided process spread  
$\bar{x} = \text{process mean}$ |
| $C_{PU} = \frac{USL - \bar{x}}{3\sigma_{within}}$ | Ratio of the difference between upper specification limit to one-sided process spread |
| $C_{PK} = \text{Min.}\{C_{PU}, C_{PL}\}$ | Takes into account the shift in the process. The measure of $C_{PK}$ relative to $C_p$ is a measure of how off-center the process is. If $C_p = C_{PK}$ the process is centered; if $C_{PK} < C_p$ the process is off-center. |
| $CC_{PK} = \frac{USL - \mu}{3\sigma_{within}}$ | $\vdots$ |
| $CC_{PK} = \text{Min.}\{(USL - \mu), (\mu - LSL)\}$ | $\vdots$ |

Note: In all the above cases, the standard deviation is the estimate of within subgroup standard deviation. As noted above, the formulas for estimating standard deviation differs from case to case. It is very important to calculate the correct standard deviation. The standard deviation formulas are discussed later.

Relationship between $C_p$ and $C_{PK}$
The index $C_p$ determines only the spread of the process. It does not take into account the shift in the process. $C_{pk}$ determines both the spread and the shift in the process.

and the relationship between $C_p$ and $C_{pk}$ is given by

$$C_{pk} = (1 - k) C_p$$

Note that $C_{pk}$ never exceeds $C_p$. When $C_{pk} = C_p$, the process is centered midway between the specification limits. Both these indexes $C_p$ and $C_{pk}$ together provide information about how the process is performing with respect to the specification limits.

**Overall Process Capability Indices (or Performance Indices)**

MINITAB also calculates the overall process capability indexes. These indexes are $P_p$, $P_{PL}$, $P_{PU}$, $P_{pk}$, and $C_m$. The formulas for calculating these indexes are similar to those of potential capabilities except that the estimate of the standard deviation is an overall standard deviation and not within group standard deviation. The formulas for the overall capability indexes refer to Table 6.18.

**Table 6.18**

<table>
<thead>
<tr>
<th>Capability Indices for Overall Process Capability</th>
<th>USL = upper specification limit</th>
<th>LSL = lower specification limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p = \frac{USL - LSL}{6 \sigma_{within}}$</td>
<td>$\sigma_{overall}$ = estimate of overall subgroup standard deviation</td>
<td></td>
</tr>
<tr>
<td>$P_{PL} = \frac{\bar{x} - LSL}{3 \sigma_{overall}}$</td>
<td>This is the performance index that does not take into account the process centering. Continued...</td>
<td></td>
</tr>
<tr>
<td>$P_{PU} = \frac{USL - \bar{x}}{3 \sigma_{overall}}$</td>
<td>Ratio of the difference between process mean and lower specification limit to one-sided process spread</td>
<td></td>
</tr>
<tr>
<td>$\bar{x} = process mean$</td>
<td>:</td>
<td></td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td></td>
</tr>
</tbody>
</table>
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\[ P_{PK} = \text{Min.}\{P_{PL}, P_{PU}\} \]

This index is the ratio of (USL-LSL) to the square root of mean squared deviation from the target. This index is not calculated if the target value is not specified. A higher value of this index is an indication of a better process. This index is calculated for the known values of USL, LSL, and the target (T).

\[
C_{pm} = \frac{USL - LSL}{(\text{tolerance})\sqrt{\frac{\sum_{i=1}^{n} (x_i - T)^2}{n - 1}}} \]

Used when USL, LSL, and target value, T are known

\[ T = \text{target value} = (USL+LSL)/2 = \mu \]

Tolerance = 6 (sigma tolerance)

\[
C_{pm} = \text{Min.}\{(T - LSL), (USL - T)\} \frac{\text{tolerance}}{2} \sqrt{\frac{\sum_{i=1}^{n} (x_i - T)^2}{n - 1}} \]

USL, LSL, and target, T are known but \( T \neq (USL+LSL)/2 \)

(Note: The above formulas are very similar to what MINITAB uses to calculate these indexes. See the MINITAB help screen for details).

In this section, we present several cases involving process capability analysis when the underlying process data are normally distributed. The process capability report and the analyses are presented for different cases.

**Examples on Process Capability**

**Example 6.3**

Calculate the capability indices - \( C_p, Cpl, Cpu, \) and \( Cpk \) for the process for which the data are given below. Interpret their meaning. Explain the difference between \( C_p \) and \( Cpk \).

\[
USL = 10.050, \ LSL = 9.950, \ \hat{\mu} = 9.999 \text{ and } \hat{\sigma} = 0.0165 \text{ as estimates of } \mu \text{ and } \sigma \text{ (} \hat{\mu} \text{ is the same as } \bar{x}).
\]

**Solution:**

\[
C_p = \frac{USL - LSL}{6\sigma} = \frac{10.050 - 9.950}{6(0.0165)} = 1.01
\]

(Note; \( \hat{\sigma} \) is the estimate of standard deviation).

\[
C_{pl} = \frac{\bar{x} - LSL}{3\sigma} = \frac{9.999 - 9.950}{3(0.0165)} = 0.99
\]
C_p=1.01 means that the process is marginally capable (just able to meet the specifications). C_p=C_pk means that the process is centered. This process is slightly off-centered.

**Difference between C_p and C_pk:** The process capability ratio or C_p does not take into account the shift in the process mean. It does not consider where the mean is relative to the specifications. C_p measures only the spread of the specifications relative to the 6-sigma spread or the process spread. C_pk on the other hand, ....

**Example 6.4**

(a) Given \( \bar{x} = 70 \) and \( \sigma = 2 \) (as estimates of \( \mu \) and \( \sigma \)), LSL =58, USL = 82. Calculate the process capability indices: C_p, C_pl, Cpu, and C_pk.

**Solution:** The problem is visually shown in Figure 6.20.

\[
C_p = \frac{USL - \bar{x}}{6\sigma} = \frac{82 - 58}{6 \times 2} = 2.0
\]

\[
C_{pl} = \frac{\bar{x} - LSL}{3\sigma} = \frac{70 - 58}{3 \times 2} = 2.0\]

\[
C_{pu} = \frac{USL - \bar{x}}{3\sigma} = \frac{82 - 70}{3 \times 2} = 2.0
\]

\[
C_{pk} = \text{Min}\{C_{pu}, C_{pl}\} = \text{Min}\{2.0, 2.0\} = 2.0
\]
(b) Calculate the capability indices for the data in part (a) if the mean has shifted from 70 to 73 (all the other values are same as in part (a)).

Example 6.5

Complete Table 6.18 by calculating the $C_p$, $C_{pk}$, and the process fallout, or the defects in parts per million (PPM) for different sigma levels. Calculate ratios $C_p$, $C_{pk}$, and the process fall out when the process is centered and when there is a shift of 1.5-sigma. Note: $C_{pk}$ is calculated based on an assumed shift of 1.5σ. Some calculations for 3σ process are shown in the table. Complete the rest of the table.

Table 6.18: Calculation of Defects (PPM) for Different Sigma Level

<table>
<thead>
<tr>
<th>Sigma Level</th>
<th>Process Centered?</th>
<th>$C_p$</th>
<th>$C_{pk}$</th>
<th>Process Fallout (Defects in PPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>±3σ</td>
<td>Yes</td>
<td>1.0</td>
<td>1.0</td>
<td>2700 PPM</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1.0</td>
<td>0.5</td>
<td>66,811 PPM</td>
</tr>
<tr>
<td>±4σ</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±4.5σ</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±6σ</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution: Calculations for ±3σ

Curve A in Figure 6.22 shows a 3σ process. Curve B shows a shift of 1.5σ in the mean. The $C_p$, $C_{pk}$, and the process fallout (in defective PPM) can be calculated as shown below.

$3\sigma$ Centered Process

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{6\sigma}{6\sigma} = 1.0$$

To calculate the process fallout or the defective, calculate $P(Z>3.0)$ using the standard normal normal curve. This can be calculated using MINITAB (see instructions in Table 6.19).
Figure 6.22: A 3-Sigma Process with a Shift of 1.5-Sigma

Table 6.19

In MINITAB, execute the following commands

Calc > Probability Distributions > Normal

Complete the Normal Distribution dialog box click the circle to the left of Cumulative probability and type the following in the boxes:

Mean 0.0
Standard deviation 1.0
Input constant 3.0

Click OK.

The session screen will show the result below.

Cumulative Distribution Function

Normal with mean = 0 and standard deviation = 1

<table>
<thead>
<tr>
<th>x</th>
<th>P(X &lt;= x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.998650</td>
</tr>
</tbody>
</table>

Using the above result, \( P(z > +3) = 1 - 0.998650 = 0.00135 \)

and, \( P(z > -3) = 0.00135 \)

Therefore, defective in a million (PPM) for a ±3σ process=(2*0.00135)*10^6 = 2700. This process fallout is for a centered process. It indicates that this process would produce 2700 defective products out of a million.

Cp, Cpk, and the process fallout (in defective PPM) for a shift of +1.5σ
Curve B shows a shift of $1.5\sigma$ in the mean. The $C_p$, $C_{pk}$, and the process fallout (in defective PPM) can be calculated as:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{6\sigma}{6\sigma} = 1.0
$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \ldots
$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} = \ldots = 0.5
$$

$$C_{pk} = \text{Min}\{C_{pu}, C_{pl}\} = \text{Min}\{0.5, 0.5\} = 0.5
$$

To calculate the process fallout or the defective, calculate $P(Z>3.0)$ using the standard normal curve with a mean of 1.5. This can be calculated using MINITAB. Follow the instructions in Table 6.19 but type 1.5 for the mean. The result is shown below.

<table>
<thead>
<tr>
<th>Cumulative Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal with mean = 1.5 and standard deviation = 1</td>
</tr>
<tr>
<td>$x$</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Using the above result, $P(z > +3) = 1 - 0.933193 = 0.0668072$

The total process fallout or defective (in PPM) for a shift of $\pm 1.5\sigma$ would be $0.066811 \times 10^6$ (6.68072+0.0003)% or, 66,811 in a million. This value is usually reported as 66,807 in the literature. Table 6.20 shows the $C_p$, $C_{pk}$, and the defective (in PPM) for $4\sigma$, $4.5\sigma$ and $6\sigma$ processes.

**Calculations for a $\pm 4\sigma$ Process with a Shift of $\pm 1.5\sigma$**

Figure 6.23 shows a 4-Sigma process with a shift of 1.5-sigma on either side. The $C_p$, $C_{pk}$, and the process fallout (in defective PPM) for this process are calculated below.
Figure 6.23: A 4-Sigma Process with a Shift of 1.5-Sigma

**When the process is centered:**

\[ C_p = \frac{USL - LSL}{6\sigma} = \frac{8\sigma}{6\sigma} = 1.33 \]

To calculate the process fallout or the defective, calculate \( P(Z > 4.0) \) using the standard normal curve. The instructions are the same as in Table 6.19 except the following — ...

| Cumulative Distribution Function
<p>| Normal with mean = 0 and standard deviation = 1 |</p>
<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(X \leq x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.999968</td>
</tr>
</tbody>
</table>

From the above table, \( P(Z > +4) = 1 - 0.999968 = 0.000032 \)

\[ C_{PU} = \frac{USL - \mu}{3\sigma} = \frac{4\sigma - 1.5\sigma}{3\sigma} = 0.83 \]

**Cp, Cpk, and the process fallout (in defective PPM) for a shift of +1.5\( \sigma \)**

Figure 6.23 shows a shift of 1.5 \( \sigma \) in the mean for a 4-sigma process. The \( C_p \), \( C_{pk} \), and the process fallout (in defective PPM) can be calculated as:
Chapter 6: Process Capability Analysis for Six Sigma

\[ C_{pk} = \text{Min.} \{ C_{pu}, C_{pl} \} = \text{Min\{..., 0.83\}} = 0.83 \]

To calculate the process fallout or the defective, calculate \( P(Z>4.0) \) using the standard normal curve with a mean of 1.5. This can be calculated using MINITAB. Follow the instructions in Table 6.19 but type a 1.5 for the mean and 4.0 in the mean box. The result is shown below.

<table>
<thead>
<tr>
<th>Cumulative Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal with mean = 1.5 and standard deviation = 1</td>
</tr>
<tr>
<td>x [ P(X \leq x) ]</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

From the above table, 
\[ P(z > 4) = 1 - 0.993790 = 0.00621 \]

Therefore, the process fallout or defective (in PPM) for a shift of \( +1.5 \sigma \) would be \( 0.00621 \times 10^6 \) or \( 6210 \). Similarly, the defective (in PPM) for a shift of \( -1.5 \sigma \) would also be \( 6210 \).

Table 6.20 shows the \( C_p \), \( C_{pk} \), and the defective (in PPM) for \( 3 \sigma, 4 \sigma, 4.5 \sigma, \) and \( 6 \sigma \) processes. Note that the defects in PPM corresponding to the \( C_{pk} \) values are calculated for only \( 1.5 \sigma \) shift in the positive direction. For \( \pm 1.5 \sigma \) shift, the process fall out values will be twice the values shown in Table 6.20. You should verify these results.

### Table 6.20: \( C_p \), \( C_{pk} \), and Process Fallout in PPM for Different Sigma Levels

<table>
<thead>
<tr>
<th>Sigma Level</th>
<th>Process Centered?</th>
<th>( C_p )</th>
<th>( C_{pk} )</th>
<th>Process Fallout (Defects in PPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pm 3\sigma )</td>
<td>Yes</td>
<td>1.0</td>
<td>1.0</td>
<td>2700</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1.0</td>
<td>0.5</td>
<td>66,807</td>
</tr>
<tr>
<td>( \pm 4\sigma )</td>
<td>Yes</td>
<td>1.33</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>( \pm 4.5\sigma )</td>
<td>Yes</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1.5</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>( \pm 6\sigma )</td>
<td>Yes</td>
<td>2.0</td>
<td>1.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>
Example 6.1- Calculating Process Capability using Control Charts

A chemical company manufactures and markets 50 lb. nitrogen fertilizer for the lawns. Due to some recent problems in their production process, overfilling and under filling in the filling bags of fertilizer have been reported. The problem was investigated and appropriate adjustments were made to the machines that are used to fill the fertilizer bags. When the process was believed to be stable, the quality supervisor collected 30 samples each of size 5. The control charts for $\bar{x}$ and R were constructed and the tests were conducted for the special or assignable causes. The process was found to be within control and no assignable causes were present. The $\bar{x}$ and R control charts for the process are shown in Figure 6.15.

Determine the process capability for this process based on the average of range value, or $\bar{R}$ reported on the R chart. Note that $\bar{R}$ - the average of all subgroup range can be used to estimate the process standard deviation.

Solution:
First, estimate the standard deviation, $\sigma$ from the given information using:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \ldots = 0.842$$

Note: $\hat{\sigma}$ is the estimate of $\sigma$. The value of $\bar{R}$ is reported in the chart for range in Figure 6.15 and $d_2$ is obtained from the table - ‘factors for computing 3 $\sigma$ control limits’ in Appendix B (Table B.3). The value of $d_2$ ....

The process capability,

$$6 \hat{\sigma} = 6(0.842) = 5.052$$
(a) The process capability can also be determined by estimating the $\hat{\sigma}$ using the average of standard deviations, $\overline{s}$ of the subgroups instead of average of the subgroup range. Here we demonstrate the calculation of process capability using the standard deviation. Figure 6.16 shows the $\overline{x}$ - $S$ - control charts for the average and standard deviation. Determine the process capability using the value of $\overline{s}$ in the control chart. First, estimate the standard deviation, $\sigma$ from the given information using:

$$\hat{\sigma} = \frac{\overline{s}}{c_4} = \ldots = 0.841$$

Note: $\hat{\sigma}$ is the estimate of $\sigma$. The value of $\overline{s}$ is reported in the chart for standard deviation ($s$)… and $c_4$ is obtained from the table - ‘factors for computing…

Figure 6.16: The $\overline{x}$ and S - control charts

The process capability: $6 \hat{\sigma} = 5.046$

Continued...
CASE 1: PROCESS CAPABILITY ANALYSIS (USING NORMAL DISTRIBUTION)

In this case, we want to assess the process capability for a production process that produces certain type of pipe. The inside diameter of the pipe is of concern. The specification limits on the pipes are $7.000 \pm 0.025$ cm. There has been a consistent problem with meeting the specification limits, and the process produces a high percentage of rejects. The data on the diameter of the pipes were collected. A random sample of 150 pipes was selected. The measured diameters are shown in the data file PCA2.MTW (column C1).

The process producing the pipes is stable. The histogram and the probability plot of the data show that the measurements follow a normal distribution. The variation from pipe-to-pipe can be estimated using the within group standard deviation or $\sigma_{\text{within}}$. Since the process is stable and the measurements are normally distributed, the normal distribution option of process capability analysis can be used.

PROCESS CAPABILITY OF PIPE DIAMETER (PRODUCTION RUN 1)

To assess the process capability for the first sample of 150 randomly selected pipes, follow the steps in Table 6.19. Note that the data are in one column (column C1 of the data file) and the subgroup size is one.

Table 6.19

<table>
<thead>
<tr>
<th>PROCESS CAPABILITY ANALYSIS</th>
<th>Open the data file PCA2.MTW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use the command sequence Stat ➤ Quality Tools ➤ Capability Analysis ➤ Normal</td>
</tr>
<tr>
<td></td>
<td>In the Data are Arranged as section, click the circle next to Single column and select or type C1 PipeDia:Run 1 in the box</td>
</tr>
<tr>
<td></td>
<td>Type 1 in the Subgroup size box</td>
</tr>
<tr>
<td></td>
<td>In the Lower spec. and Upper spec. boxes, type 6.975 and 7.025 respectively</td>
</tr>
<tr>
<td></td>
<td>Click OK</td>
</tr>
<tr>
<td></td>
<td>Click the Options tab on the upper right corner</td>
</tr>
<tr>
<td></td>
<td>Type 7.000 in the Target (adds Cpm to table) box</td>
</tr>
<tr>
<td></td>
<td>In the Calculate statistics using box a 6 should show by default</td>
</tr>
<tr>
<td></td>
<td>Under Perform Analysis, Within subgroup analysis and Overall analysis boxes should be checked (you may uncheck the analysis</td>
</tr>
</tbody>
</table>
not desired)
Under **Display**, select the options you desire (some are checked by default)
Type a title if you want or a default title will be provided
Click **OK** in all dialog boxes.

The process capability report as shown in Figure 6.11 will be displayed.

![Process Capability of PipeDia: Run 1](image)

**Figure 6.11: Process Capability Report of Pipe Diameter: Run 1**

**INTERPRETING THE RESULTS**

1. The upper left box reports the process data including the lower specification limit, target, and the upper specification limit. These values were provided by the program. The calculated values are the process sample mean and the estimates of within and overall standard deviations.
Chapter 6: Process Capability Analysis for Six Sigma

2. The report in Figure 6.11 shows the histogram of the data along with two normal curves overlaid on the histogram. One normal curve (with a solid line) ........

3. The histogram and the normal curves can be used to check visually if the process data are normally distributed. To interpret the process capability, the normality assumption must hold. In Figure 6.11, ......

4. There is a deviation of the process mean (7.010) from the target value of 7.000. Since the process mean is greater than the target value, the pipes produced by this process exceed the upper specification limit (USL). A significant percentage of the pipes are outside of .......................

5. The **potential or within process capability** and the **overall capability** of the process is reported on the right hand side. For our example, the values are

<table>
<thead>
<tr>
<th>Potential (Within) Capability</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp</td>
<td>0.86</td>
</tr>
<tr>
<td>CPL</td>
<td>1.21</td>
</tr>
<tr>
<td>CPU</td>
<td>0.50</td>
</tr>
<tr>
<td>Cpk</td>
<td>0.50</td>
</tr>
<tr>
<td>CCpk</td>
<td>0.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overall Capability</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pp</td>
<td>0.88</td>
</tr>
<tr>
<td>PPL</td>
<td>1.25</td>
</tr>
<tr>
<td>PPU</td>
<td>0.51</td>
</tr>
<tr>
<td>Ppk</td>
<td>0.51</td>
</tr>
<tr>
<td>Cpm</td>
<td>0.59</td>
</tr>
</tbody>
</table>


Chapter 6: Process Capability Analysis for Six Sigma

6. The value of \( Cp=0.86 \) indicates that the process is not capable (\( Cp < 1 \)). Also, \( Cpk = 0.50 \) is less than \( Cp=0.86 \). This means that the process is off-centered. Note that when \( Cpk = Cp \) then the process …..

7. \( Cpk=0.50 \) (less than 1) is an indication that an improvement in the process is warranted.

8. Higher value of \( Cpk \) indicates that the process is meeting the target with minimum process variation. If the process is off-centered, \( Cpk \) value is smaller compared to \( Cp \) even ….............

9. The overall capability indexes or the process performance indexes \( Pp, PPL, PPU, Ppk, \) and \( Cpm \) are also calculated and reported. Note that these indexes are based on the estimate of overall standard deviation……...

10. \( Pp \) and \( Ppk \) have similar interpretation as \( Cp \) and \( Cpk \). For this example, note that \( Cp \) and \( Cpk \) values (0.86 and 0.50 respectively) are very close to \( Pp \) and \( Ppk \) (0.88 and 0.51). When \( Cpk \) equals \( Ppk \) then the within subgroup standard deviation is ….

11. The index \( Cpm \) is calculated for the specified target value. If no target value is specified, \( Cpm \) …..

12. ….............For this process, \( Pp = 0.88 \), \( Ppk = 0.51 \), and \( Cpm = 0.59 \). A comparison of these values indicates that the process is off-center.

13. The bottom three boxes report observed performance, expected within performance, and expected overall process performance in parts per million (PPM). The observed performance box in Figure 6.11 shows the following values:

<table>
<thead>
<tr>
<th>Observed Performance</th>
<th>PPM &lt; LSL</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPM &gt; USL</td>
<td>53333.33</td>
<td></td>
</tr>
<tr>
<td>PPM Total</td>
<td>53333.33</td>
<td></td>
</tr>
</tbody>
</table>

27
This means that the number of pipes below the lower specification limit (LSL) is zero; that is, ..... 

The expected "within" performance is based on the estimate of within subgroup standard deviation. These are the average number of parts below and above the specification limits in parts per million. The values are calculated using the following formulas:

\[
P \left[ z < \frac{(L_{SL} - \bar{x})}{\sigma_{within}} \right] \times 10^6 \quad \text{for the expected number below the LSL}
\]

\[
P \left[ z > \frac{(U_{SL} - \bar{x})}{\sigma_{within}} \right] \times 10^6 \quad \text{for the expected number above the USL}
\]

Note that \( \bar{x} \) is the process mean.

For this process, the Expected Within Performance measures are

<table>
<thead>
<tr>
<th>Exp. Within Performance</th>
<th>PPM &lt; LSL</th>
<th>PPM &gt; USL</th>
<th>PPM Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>134.53</td>
<td>66163.57</td>
<td>66298.10</td>
</tr>
</tbody>
</table>

The above values show the ....

14. The Expected Overall Performance is calculated using similar formulas as in within performance, except the estimate of standard deviation is based on overall data.

For this process, the Expected Overall Performance measures are

<table>
<thead>
<tr>
<th>Exp. Overall Performance</th>
<th>PPM &lt; LSL</th>
<th>PPM &gt; USL</th>
<th>PPM Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>92.20</td>
<td>61212.82</td>
<td>61305.02</td>
</tr>
</tbody>
</table>

These values are based on ..... continued...
CASE 3: PROCESS CAPABILITY OF PIPE DIAMETER (PRODUCTION RUN 3)

CASE 4: PROCESS CAPABILITY ANALYSIS OF PIZZA DELIVERY

A Pizza chain franchise advertises that any order placed through a phone or the internet will be delivered in 15 minutes or less. If the delivery takes more than 15 minutes, there is no charge and the delivery is free. This offer is available within a radius of 3 miles from the delivery location.

In order to meet the delivery promise, the Pizza chain has set a target of 12 ± 2.5 minutes....

Using the 100 delivery times (shown in Column 1 of data file PAC3.MTW), a process capability analysis was conducted. To run the process capability, follow the instructions in Table 6.20. The process capability report is shown in Figure 6.14.
CASE 6: PERFORMING PROCESS CAPABILITY ANALYSIS WHEN THE PROCESS MEASUREMENTS DO NOT FOLLOW A NORMAL DISTRIBUTION (NON-NORMAL DATA)

The process capability report is shown in Figure 6.21.
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Process Capability of Non-normal Data Using Johnson Transformation

The other way of determining the process capability of non-normal data is to use a distribution fit approach. In cases where data are not normal, fit an appropriate distribution and use that distribution—rather than a normal distribution—to determine the process capability. We will illustrate the method using an example.

Figure 6.26: Probability Plots for Selected Distribution
Another option available for process capability analysis is *process capability six-pack*. The process capability using this option displays:

- A chart (or individual chart for subgroup means),
- An R-chart (S chart for a subgroup size greater than 8),
- A run chart or moving range chart,
- A histogram,
- A normal probability plot of the process data to check the normality, and
- The between/within statistics and overall capability indexes.

### Figure 6.34: Process Capability Six-pack of Shaft Diameter

### INTERPRETING THE RESULTS

The report shows the control charts for Xbar and R. The tests for special causes are conducted and reported on the session screen. No special causes were found indicating that the process is stable and in control. The capability histogram shows that the....................

*Chapter 6 of Six Sigma Volume 1 contains detailed analysis and interpretation of process capability analysis with data files and step-wise computer instructions for both normal and non-normal data.*

To buy chapter 6 or Volume I of Six Sigma Quality Book, please click on our products on the home page.