Chapter 5

Quality Tools for Six Sigma

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Box Plot
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  - Example 2: A Run Chart with Subgroup Size Greater than 1
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- Example 21: Determine a Machine-to-Machine, Time-to-Time variation
- Part-to-Part Variation in a Production Run using Multi-vari and Other Plots

Symmetry Plot
Summary and Applications

This chapter explains the graphical techniques that have been applied as problem-solving tools in quality control. Many graphs and charts are helpful in detecting and solving quality problems. Some of the graphical techniques in this chapter are described in previous chapters. In this chapter we describe and analyze the graphs and charts as they relate to quality control. We also provide examples and specific situations in which these graphical techniques are used in detecting and solving quality problems.

Some examples from the chapter are presented below. The book provides step-wise instructions with data files for each case.
Histograms

In quality, histograms are useful in

- detecting process problems-including a shift in the process
- evaluating process capability (ability of the process to be within its specification limits)
- determining how well centered the process is or how close the data values are to the target value, and
- determining the process variation.

**Examples**

Figure 5.1(a) Most values within specification, some assignable causes may be present

Figure 5.1(c) Process variation has reduced compared to (a): a shift to the right
Figure 5.1(e) The process has shifted to the left; products out of specification (may be calibration problem)

Figure 5.1(f) Process shift to the left; more variation compared to (e)

Figure 5.1 (i) Process stable and close to the target (desirable)

The above histograms are useful in examining the possible problems and the causes behind them.

**Evaluating Process Capability using Histograms**

Histograms can also be used to assess the process capability (the ability of a process to be
within its specifications). The process capability can be determined once the process is stable ....... The next chapter provides the measures of process capability and their relationship to six sigma quality.

### Run Charts

A run chart is used in Quality Control to analyze the data either in the development stage of a product or before the state of statistical control.

A run chart can be used to determine if the process is running in a state of control or if special or assignable causes are influencing the process, thereby making the process out of control.

#### Example 4: Run Chart Showing a Stable Process, a Shift, and a Trend

![A Run Chart Showing a Stable Process](image-url)

<table>
<thead>
<tr>
<th>Observation</th>
<th>Process 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>3.5</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
</tr>
<tr>
<td>8</td>
<td>2.5</td>
</tr>
<tr>
<td>9</td>
<td>3.5</td>
</tr>
<tr>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>11</td>
<td>2.5</td>
</tr>
<tr>
<td>12</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- **Number of runs about median**: 53
- **Expected number of runs**: 46.00000
- **Longest run about median**: 4
- **Approx P-Value for Clustering**: 0.93111
- **Approx P-Value for Mixture**: 0.06889
- **Number of runs up or down**: 56
- **Expected number of runs up or down**: 59.66667
- **Longest run up or down**: 5
- **Approx P-Value for Trends**: 0.17721
- **Approx P-Value for Oscillation**: 0.82279
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A Run Chart Showing a Trend

Run Chart Showing a Trend

A Run Chart Showing a Shift in the Process

A Run Chart Showing a Shift

A run chart is a quick, easy, and economical way to detect process problems. In the initial stages of a process, or for a new process, run charts provide opportunities for improvement without the implementation of control charts.
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Pareto Chart

A Pareto chart is very similar to a histogram or a frequency distribution of attribute data where the bars are arranged by categories from largest to smallest with a line that shows the cumulative percentage and count of the bars. This chart is widely used in Quality Control to analyze the defect data. Through this chart, the defects that occur most frequently can be identified quickly and easily. This helps to focus the improvement efforts on the largest percentage of defects. A Pareto chart does not identify the most important category; it identifies the categories that occur most frequently. Pareto charts are also used widely in non-manufacturing applications.

There are several variations of the Pareto chart. MINITAB provides several options to construct this chart. The data for the chart can be entered in several ways.

**Example 6: Pareto Chart with Cumulative Percentage (Second Option)**

Pareto Chart of Defects in Machined Parts

<table>
<thead>
<tr>
<th>Failure Cause</th>
<th>Count</th>
<th>Percent</th>
<th>Cum %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incorrect Dimensions</td>
<td>84</td>
<td>27.8</td>
<td>27.8</td>
</tr>
<tr>
<td>Damaged Parts</td>
<td>55</td>
<td>18.2</td>
<td>46.0</td>
</tr>
<tr>
<td>Machining Errors</td>
<td>40</td>
<td>13.2</td>
<td>59.3</td>
</tr>
<tr>
<td>Internal Flaws</td>
<td>33</td>
<td>10.9</td>
<td>70.2</td>
</tr>
<tr>
<td>Measurement Errors</td>
<td>25</td>
<td>8.3</td>
<td>78.5</td>
</tr>
<tr>
<td>In-use Failures</td>
<td>21</td>
<td>7.0</td>
<td>85.4</td>
</tr>
<tr>
<td>Surface Finish Errors</td>
<td>17</td>
<td>5.6</td>
<td>91.1</td>
</tr>
<tr>
<td>Damaged Errors</td>
<td>15</td>
<td>5.0</td>
<td>96.0</td>
</tr>
<tr>
<td>Incorrect Dimensions</td>
<td>12</td>
<td>4.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Pareto Chart of Defect Data with No Cumulative Points Plotted
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Example 8: Pareto Chart by Variable (Fourth Option)

The data file `PARETOCHART.MTW` shows the types of failures in manufactured parts and the shifts they produced (columns C3 and C4). Suppose we want to construct Pareto charts for each of the three shifts. This will help visualize the defect data for each shift separately and analyze the causes of defects. The Pareto charts for each Shift vs. Types of Failure are shown below.
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Pareto Chart of Defects vs. Shift

Pareto Chart of Defects vs. Shift (all shifts plotted on one graph)

Cause-and-Effect Diagrams or Fishbone Diagrams

A cause-and-effect diagram is a very useful tool in establishing the relationship between the causes and their effects. Once a problem has been identified, it is necessary to analyze potential cause or causes of the problem. In such cases, the cause-and-effect diagram is a very helpful tool. To construct a cause-and-effect diagram:

(a) Identify the problem or effect

(b) Identify the cause or possible causes of the problem through study or experience

: :

(f) Take necessary action to solve the problem.
Example 9: Cause-and-effect Diagram (1)

The problem is to study the possible causes of a high percentage of scrap and rework produced. The problem (effect) is identified and possible causes are to be explored using a cause-and-effect diagram. The data file CAUSE.MTW shows six major causes for scrap and rework (Machines, Materials, Methods, Measurement, Personnel, and Environment) in columns C1-C6. Each major cause is further classified into categories that are shown below each major cause (see columns C1-C6 of the data file CAUSE.MTW). The problem or the effect is typed in the **Effect** box when the dialog box is displayed.

Note: MINITAB provides six major areas which are considered the main causes of problems. These are Personnel or people, methods, materials, machines, measurements, and environment. These are provided as the default causes for the cause-and-effect diagram. However, these defaults can be changed as desired to create different cause categories.

![A Cause-and-Effect Diagram of Scrap and Rework](image)

A Cause-and-Effect Diagram of Rework and Scrap
**Example 10: Cause-and-effect Diagram (2)**

**Major Causes of Production Problems**

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Material</th>
<th>Personnel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>Material quality</td>
<td>Training</td>
</tr>
<tr>
<td>Equipment</td>
<td>Supplier</td>
<td>Lack of experience</td>
</tr>
<tr>
<td>Gage problem</td>
<td>Many suppliers</td>
<td>Work instructions</td>
</tr>
<tr>
<td>Wrong gaging</td>
<td>Lead Time</td>
<td>Absenteeism</td>
</tr>
<tr>
<td>Observer errors</td>
<td>Delivery</td>
<td>Poor tools</td>
</tr>
<tr>
<td>Lack of training</td>
<td>Internal flaws</td>
<td>Lack of motivation</td>
</tr>
<tr>
<td>Fatigue</td>
<td>External flaws</td>
<td>Communication</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Environment</th>
<th>Methods</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work conditions</td>
<td>Analysis</td>
<td>Pressure</td>
</tr>
<tr>
<td>Distractions</td>
<td>Design problem</td>
<td>Production</td>
</tr>
<tr>
<td>Noise</td>
<td>Technology</td>
<td></td>
</tr>
<tr>
<td>Heat</td>
<td>No automation</td>
<td></td>
</tr>
<tr>
<td>Dirt</td>
<td>Poor skills</td>
<td></td>
</tr>
<tr>
<td>Dust</td>
<td>Instructions</td>
<td></td>
</tr>
<tr>
<td>Humidity</td>
<td>Process planning</td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>Work methods</td>
<td></td>
</tr>
</tbody>
</table>

**A Cause-and-Effect Diagram of Production Problems**

The Cause-and-Effect Dialog Box for Cost of Poor Quality

**Costs of Poor Quality (COPQ)**

**Prevention**
- Improve Quality eng
- Improve Manufacturing eng
- Improving design
- Cost of quality planning
- Cost of quality training
- Special equipments to control
- Building supplier relations
- Variability analyses
- Cost of skilled quality professional
- Cost of field audits
- Cost of product audits
- Test and inspection records
- Cost of inspection and testing
- Cost of maintaining test
- Cost of maintaining materials
- Inspection Supervisors

**Internal**
- Scrap
- Rework
- Repair
- Retest
- Failure analysis
- Downtime
- Cost of yield losses
- Downgrading products
- Loss of future business
- Loss of market share
- Customer dissatisfaction
- Loss of goodwill
- Paperwork processing
- Corrective action
- Settlement costs
- Legal services and costs
- Cost of testing
- Recalls
- Customer complains
- Returned
- Warranty charges
Summary and Applications of Quality Tools

<table>
<thead>
<tr>
<th>Type of Chart/Graph</th>
<th>Applications</th>
<th>Number of Variables Plotted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run Chart</td>
<td></td>
<td>One variable plotted</td>
</tr>
<tr>
<td>Stem-and-leaf Plot</td>
<td>:</td>
<td>One variable plotted</td>
</tr>
<tr>
<td>Box-plot</td>
<td>Plot of five measures: the minimum, first quartile, median, third quartile, and the maximum. Displays the distribution of underlying data.</td>
<td>One variable plotted</td>
</tr>
<tr>
<td>Pareto Chart</td>
<td>The chart shows (in descending order) the contribution of the vital few versus the trivial many. Used to identify the few problems, causes, sources, or defects that should be considered first in the problem-solving process.</td>
<td>One variable divided into different categories</td>
</tr>
</tbody>
</table>

All the above graphs—except the cause-and-effect diagram—are used to summarize or describe one characteristic at a time, and therefore, describe univariate data or measurements made on one variable.

Bivariate Data: Measuring and Describing Two Variables

Bivariate data consist of two variables. A scatter plot is usually used to describe the relationship between two variables. MINITAB provides several options for scatter plots, which were explained in chapter 3. Here we consider some more applications of scatter plots.

Scatter Plots

In a scatterplot, two variables (usually denoted by X and Y) are examined. A sample of n observations is collected and the bivariate data (x1, y1), (x2, y2), ..., (xn, yn) are plotted. The plot is used to extract the relationship between x and y. Several of these plots were described in Chapter 3.

Sometimes it is useful to plot histograms, box plots, or dot plots of x and y variables.
along with the scatter plot. These plots are helpful in determining the relationship between $x$ and $y$, and their distributions.

**Example 12: Scatterplots with Histograms, Box plots, and Dot plots**

Scatterplot with Histograms of $x$ and $y$ Variables

Scatterplot with Box Plots of $x$ and $y$ Variables
Example 15: Scatterplot Showing a Nonlinear Relationship between X and Y

The plot shows a strong relationship between x and y. The equation relating the temperature and the yield can be very useful in predicting the maximum yield or optimizing the yield of the process. Note that the equation of the fitted curve is
\[ y = -1022 + 320.3 \cdot x - 1.054 \cdot x^2. \]

In this equation, y is the yield and x is the temperature. The equation relating the two variables is written on the plot.

Multi-Vari Charts and Other Plots Useful to Investigate Relationships Before Running the Analysis of Variance

This section presents several plots that can be used to present the information in the form of graphs and charts in the preliminary stages of data analysis. These plots can be very helpful in visualizing the data, and provide invaluable information for a formal analysis of variance. The graphs and chart include
- Multi-vari Chart
- Box plot
- Main effects plot
- Interaction plot
- Marginal plot

Some of these plots—for example the box plot and marginal plot—were explained earlier. Here we explain how to construct all of the above plots. Next, we will analyze the resulting graphs to get further insights. The analyses can be helpful in determining the type of statistical tests to be used for the data.
Example 17: A Multi-vari Chart for a Two-factor Design

The marketing manager of a departmental store chain is interested in studying the effect of store location and store size on the profit. Four different locations (A, B, C, and D) and three different store sizes (small, medium, and large) were selected for the study. For each store location a random sample of two stores of each size was selected and the monthly profits (in thousands of dollars) were recorded. Table 5.18 shows the data. Note that this problem can be formulated as a two-factor ANOVA (analysis of variance) with **Store Size** as Factor A with three levels (small, medium and large); **Store Locations** as Factor B with four levels (A, B, C, and D), and the **Profit** as the response variable.

<table>
<thead>
<tr>
<th>Store Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store Size</td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Medium</td>
</tr>
<tr>
<td>Large</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Totals</td>
</tr>
<tr>
<td>Means</td>
</tr>
</tbody>
</table>

Multi-Vari Chart

This chart displays the mean values at each factor level for every factor. For our example, there are two factors: store size and store location. The multi-vari plot will display the average profit for each level of store size: small (1), medium (2), large (3) and each level of store location: A(1), B(2), C(3), and D(4).

The multi-vari chart is shown below.
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A Multi-Vari Chart for Profit by Store Location and Store Size

In this figure, we have plotted the two factors. The solid lines connect the means of factor B (store location) levels (at each level of factor A, store size). The dotted line connects the means of factor A (store size) levels.

Main Effects Plot

Interaction plots are used to assess the interaction effects between the variables. For our example, the response variable is Profit, and there are two factors of interest; Store Size and Store Location. Our objective is to determine how profit is affected by store size and location and also by their combination. In cases where there are two or more factors are involved, the interaction effects are critical.
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Interaction Plot of Profit vs. Store Size and Store Location

The presence or absence of interaction can be determined from the above interaction plot.

A Multi-Vari Chart for Strength by Alloy Type and Thickness

A Multi-Vari Chart for Strength by Thickness and Alloy Type
Both the plots above show that alloy type 2 and thickness 2 has the maximum strength. Also, there is an indication of interaction between the alloy type and thickness because the response variable is not a linear function of the combinations of the levels of the two factors.

Interaction Plot of Strength by Thickness and Strength by Alloy Type

**Example 19: Multi-vari Chart for a Three-factor Design**

Multi-Vari Chart for Strength by Grain Size, Temperature, and Material Type

In this figure:
- Factor 1 is Grain Size, Factor 2 is Temperature, Factor 3 is Material Type
- Black lines connect the means of factor 1 levels (at each combination between factor 2 and 3 levels).
• Red lines (dotted lines) connect the means of factor 2 levels (at each level of factor 3).
• A green line (the solid line from the center of columns 1, 2, and 3) connects the means of factor 3 levels.

For 3 and 4 factors, the multi-vari chart is paneled. The panel variable is the 3rd factor for a 3 factor chart.

Interaction Plot: Material Type & Grain Size, Material Type & Temperature, and Grain Size & Temperature

Summary and Applications of Plots

<table>
<thead>
<tr>
<th>TYPE OF CHART/GRAPH</th>
<th>APPLICATIONS</th>
<th>NUMBER OF VARIABLES PLOTTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scatterplot</td>
<td>In a scatterplot, two variables usually denoted by X and Y are examined. A sample of n observations is collected and the bivariate data (x1, y1), (x2, y2),..., (xn, yn) are plotted. The plot is used to extract the relationship between x and y. Types of Scatterplot: (a) scatterplot with histograms, box plots, or dot plots of x and y variables along with the x,........ (b) Scatterplot with Fitted Line or Curve: these scatterplots show if the relationship between x and y is linear or nonlinear. The plot is also used to determine the correlation or degree of association ........</td>
<td>Two variable plotted (Bivariate data)</td>
</tr>
<tr>
<td>Marginal Plot</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Type of Chart/Graph</th>
<th>Applications</th>
<th>Number of Variables Plotted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-vari Chart</td>
<td>Mult-vari charts can be used as clue generation techniques. They can be used in preliminary stages of data analysis to investigate the relationships between the main factors and their interactions. Two-to four factor analysis of variance data can be plotted using multi-vari chart....... : In a discrete manufacturing environment these charts can be used to detect part-to-part variation, within part variation, and also variation between time periods etc. In a continuous manufacturing, the chart can be used to detect shift-to-shift, day-to-day or week-to-week variation in the response variable.</td>
<td>Up to four factors can be plotted using MINITAB</td>
</tr>
<tr>
<td>Interaction Plot</td>
<td>Interaction plots are used to assess the interaction effects between the variables. In the initial stages of data analysis, interaction plots can be used to see the interactions between variables. These plots can visually show the interaction effects without formally running the analysis of variance (ANOVA). However, the interaction plot may be misleading in .....................</td>
<td>Interaction plot matrix of several factors can be generated using MINITAB.</td>
</tr>
<tr>
<td>Symmetry Plot</td>
<td>Symmetry plots are a quick and easy way to check if the data follows a symmetrical distribution. The symmetry plot is generated by first calculating the median of the data and then forming the first pair of values that are closest to the median: one above and one below the median. The second pair is formed using the two values that .......</td>
<td>One variable plotted at a time</td>
</tr>
</tbody>
</table>

The detailed treatment of all the above plots with computer instructions and data files can be found in Chapter 5.

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